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GAMES OF STRATEGIC COMPLEMENTARITIES: AN APPLICATION TO BAYESIAN GAMES

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Games of Strategic Complementarities: An Application to Bayesian Games *

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Abstract

This paper provides an introduction to the theory of games of strategic complementarities, considers Bayesian games, and provides an application to global games.

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1 Introduction

Games of strategic complementarities are those in which any player increases his action in response to an increase in the level of actions of rivals. The complementarity idea for an individual agent goes back to Edgeworth (1881) with the notion that the marginal value of an action is increasing in the level of other actions available to the agent. Many games display strategic complementarities, including those with search, network externalities, oligopoly interaction, or technology adoption and patent races. Additional examples include coordination failures in macroeconomics and financial markets, as well as cumulative processes in the presence of complementarities.

The theory of monotone comparative statics and supermodular games (Topkis (1978, 1979), Vives (1985, 1990) and Milgrom and Roberts (1990)) provides the set of tools to deal with complementarities. This theory, in contrast to classical convex analysis, is based on order and monotonicity properties on lattices. The analysis of monotone comparative statics provides conditions under which solutions to optimization problems change monotonically with a parameter. The theory of supermodular games exploits order and monotonicity properties to ensure that the best response of a player to the actions of rivals is increasing in their level. The approach is powerful. In the class of supermodular games:

- Very general strategy spaces, including indivisibilities and functional spaces such as those arising in dynamic or Bayesian games, are allowed.
- Equilibrium in pure strategies exists (without requiring quasiconcavity of payoffs).
- The equilibrium set has an order structure with extremal elements (allowing a global analysis of the set).
- There is an algorithm to compute extremal equilibria, which bound also the rationalizable set, and the equilibrium set has nice stability properties.
- Monotone comparative statics results are obtained with minimal assumptions and unambiguous predictions are possible even in the presence of multiple equilibria.

The last point is particularly relevant since coordination failures and multiple equilibria are typical in the presence of complementarities. Bank or debt runs, currency crises, low activity equilibria, adoption externalities, and development traps are some examples.

In this paper I will provide a brief intuitive account of the theory based on a very simple framework and I will develop some connections with games of incomplete information and more in particular, global games. The reader is referred to Vives (2005) for a more detailed account of the summary presented in this paper, and to Topkis (1998) and Vives (1999) for general treatments of the tools and results associated to games of strategic complementarities.

I introduce a simple class of games in Section 2 to highlight the main results of the theory.

Section 3 deals with supermodular games and Section 4 with Bayesian games. Section 5 provides an application to global games.

2 The basic ideas in a simple framework

In a game of strategic complementarities –the term was coined in Bulow et al. (1983)– each player responds to an increase in the strategies of the rivals with an increase in his own strategy.

Consider a symmetric game with a continuum of players. The payoff to player i is given by $\pi(a_i, \widetilde{a}; \theta)$ where $a_i \in [0,1]$ is the action of the player, $\widetilde{a}$ is the average or aggregate action, and θ is a payoff-relevant parameter. Suppose that π is smooth in all arguments with $\partial^2\pi / (\partial a_i)^2 < 0$, and let $r(\widetilde{a}; \theta)$ be the best response of an individual player to the aggregate action $\widetilde{a}$. Then any equilibrium will be symmetric, with $\widetilde{a} = a_i = a$ and fulfilling $r(a; \theta) = a$. For interior solutions, we have that $\partial\pi/\partial a_i = 0$ and $r(\cdot)$ is continuously differentiable with

$$r'(\widetilde{a}) \equiv \frac{\partial r(\widetilde{a}; \theta)}{\partial \widetilde{a}} = -\left(\frac{\partial^2 \pi}{\partial a_i \partial \widetilde{a}}\right) / \left(\frac{\partial^2 \pi}{(\partial a_i)^2}\right).$$

Therefore,

$$\text{sign}\{r'(\widetilde{a})\} = \text{sign}\left\{\frac{\partial^2 \pi}{\partial a_i \partial \widetilde{a}}\right\}$$

and the best reply is increasing if $\partial^2\pi/\partial a_i \partial\widetilde{a} \geq 0$. Suppose also that $\partial^2\pi/\partial a_i \partial\theta \geq 0$, so that an increase in θ increases the marginal profit of the action of a player and consequently his best response $r(\cdot)$.

A classical example of the model are adoption, search or aggregate demand externalities (see, e.g., Diamond (1981), Dybvig and Spatt (1983), or Cooper and John (1988)). The action a_i may be the effort of trader i in looking for a partner or the adoption level of a technology. The benefit is proportional to own effort/adoption level and is increasing in the aggregate effort/adoption level $\widetilde{a}$ of others

$$\pi(a_i, \widetilde{a}; \theta) = \theta a_i g(\widetilde{a}) - C(a_i),$$

with $\theta > 0$ and where $g(\cdot)$ and the cost of effort/adoption $C(\cdot)$ are increasing nonnegative functions. In this case, $\partial^2\pi/\partial a_i \partial\widetilde{a} = \theta g'(\widetilde{a}) \geq 0$ and $\partial^2\pi/\partial a_i \partial\theta = g(\widetilde{a}) \geq 0$.

In games of strategic complementarities we have typically multiple Pareto–rankable equilibria. For example, suppose that g has an S-shaped function and $C'(a) \equiv a$. Then $r(\widetilde{a}) = \theta g(\widetilde{a})$ and there may be three equilibria given by the solutions (i.e., $\underline{a}, \widehat{a}$, and $\overline{a}$) to $\theta g(a) = a$ as depicted in the lower branch of Figure 1. Larger activity equilibria dominate equilibria with less activity since there are positive spillovers: $\partial\pi/\partial\widetilde{a} = \theta a_i g'(\widetilde{a}) \geq 0$.

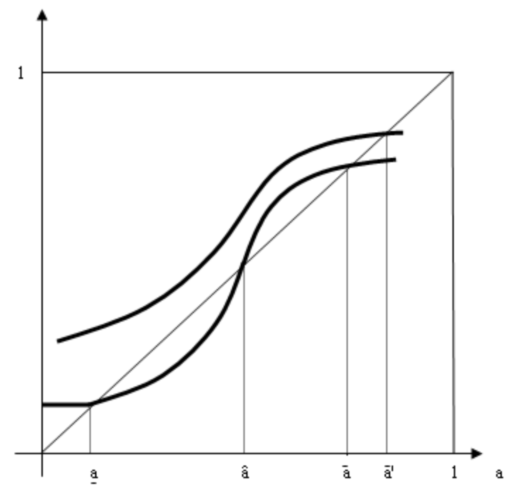

Figure 1

Monopolistic competition provides another illustration. Here a_i is the price of the variety produced by firm i, $\widetilde{a}$ the average price in the market and θ, say, the (common) marginal cost. We have then

$$\pi\left(a_i, \widetilde{a}; \theta\right) = \left(a_i - \theta\right) D\left(a_i, \widetilde{a}\right)$$

with $D\left(\cdot\right)$ the demand function. It is easy to check that $\partial^2 \log \pi / \partial a_i \partial \widetilde{a} = \partial^2 \log D / \partial a_i \partial \widetilde{a} > 0$ in many demand systems –where the elasticity of demand for product i is decreasing in the average price. Under this condition $r'\left(\widetilde{a}\right) > 0$ because best replies are invariant to an increasing transformation of the payoffs, such as the logarithm. Furthermore, $\partial^2 \pi / \partial a_i \partial \theta = -\partial D / \partial a_i > 0$ with downward sloping demand. The monopolistic competition model has been used extensively in the growth, development, regional, and international trade literatures to generate complementarities and multiplier processes (see Matsuyama (1995) for a survey).

Several properties of the equilibria in the examples are easy to derive:

1. *Existence and order structure of the equilibrium set.* There exists a largest ($\overline{a}$) and a smallest ($\underline{a}$) equilibrium.

2. *Multiple equilibria and welfare.* There are multiple equilibria when strategic complementarities are sufficiently strong: $r'\left(a\right) > 1$ for some candidate equilibrium such as point $\widehat{a}$ in Figure 1. Equilibria can be Pareto ranked when there are positive spillovers.

3. *Stability and rationalizability.* Best–reply dynamics starting at $a = 0$ (resp,. $a = 1$) converge to $\underline{a}$ (resp., $\overline{a}$). Similarly, iterated elimination of strictly dominated strategies defines two

sequences that converge, respectively, to $\underline{a}$ and $\overline{a}$. This means that rationalizable strategies will lie in the interval $[\underline{a}, \overline{a}]$, and if the equilibrium is unique then the game will be dominance solvable (and globally stable).

4. *Comparative statics and multiplier effects*. An increase in the parameter θ leads to an increased action in equilibrium, via out-of-equilibrium best-reply dynamics, and this increase will be over and above the direct effect of the increase in the parameter. Indeed, increasing θ will move $r(\cdot)$ upward (as in Figure 1), and the equilibrium level of a will increase. Starting at $a = \overline{a}$, the direct effect will lead us to $r(\overline{a}) > \overline{a}$ and the full equilibrium impact to $\overline{a}' > \overline{a}$. At a stable equilibrium $a^* : r' < 1$,
$$\frac{\partial a^*}{\partial \theta} = \frac{\partial r/\partial \theta}{1 - r'} > \frac{\partial r}{\partial \theta}$$
provided that $r' > 0$. The multiplier is at work whether we focus on extremal (or stable) equilibria, or consider best–response dynamics after the perturbation. This is so even starting at an unstable equilibrium, or at an equilibrium that disappears once θ increases (e.g. in Figure 1 the unstable equilibrium $\widehat{a}$ disappears with the increase in θ, moving $r(\cdot)$ upward, and best–reply dynamics lead to the new equilibrium $\overline{a}'$).

3 Supermodular games

Consider the game $(A_i, \pi_i; i \in N)$, where N is the set of players, $i = 1, ..., n$; A_i is the strategy set and π_i the payoff of player $i \in N$ (defined on the cross product of the strategy spaces of the players A). Let $a_i \in A_i$ and $a_{-i} \in \prod_{j \neq i} A_j$ (i.e., we denote by a_{-i} the strategy profile $(a_1, ..., a_n)$ excluding the ith element). In Euclidean space a supermodular game is one where for each player i, the strategy set A_i is a compact rectangle (or "box"), the payoff function π_i is continuous and fulfills two complementarity properties:

- Supermodularity in own strategies (π_i is *supermodular* in a_i): the marginal payoff to any strategy of player i is increasing in the other strategies of the player.
- Strategic complementarity in rivals' strategies (π_i has *increasing differences in* (a_i, a_{-i})): the marginal payoff to any strategy of player i is increasing in any strategy of any rival player.

In a more general formulation of a supermodular game, strategy spaces need only be "complete lattices" and the continuity requirement can be weakened. Supermodularity and increasing differences can also be weakened to define an "ordinal supermodular" game, relaxing supermodularity to the weaker concept of quasi–supermodularity and increasing differences to a single–crossing property (see Milgrom and Shannon (1994)). Such properties –unlike supermodularity and increasing differences– have no differential characterization and need not be preserved under addition or partial maximization operations.

In a supermodular game, very general strategy spaces can be allowed. These include indivisibilities as well as functional strategy spaces, such as those arising in dynamic or Bayesian games. Regularity conditions such as concavity and interior solutions can be dispensed with.

Results 1-4 in the simple framework, described in the previous section, generalize in a supermodular game:

1' There always exist extremal equilibria: a largest $\overline{a}$ and a smallest element $\underline{a}$ of the equilibrium set. If the game is symmetric the extremal equilibria are symmetric.

2' Multiple equilibria are common. If the game displays positive spillovers, i.e. the payoff to a player is increasing in the strategies of the other players, then the largest equilibrium point is the Pareto best equilibrium, and the smallest one the Pareto worst.

3' Simultaneous best-reply dynamics approach the "box" $[\underline{a}, \overline{a}]$ defined by the smallest and the largest equilibrium points of the game; and converge monotonically downward (upward) to an equilibrium starting at any point in the intersection of the upper (lower) contour sets of the largest (smallest) best replies of the players. The extremal equilibria $\underline{a}$ and $\overline{a}$ correspond to the largest and smallest serially undominated strategies. Therefore, if the equilibrium is unique then the game is dominance solvable (and globally stable).

4' If $\pi_i(a_i, a_{-i}; \theta)$ has increasing differences in (a_i, θ) for each i then with an increase in θ: (i) the largest and smallest equilibrium points increase; and (ii) starting from any equilibrium, best–reply dynamics lead to a larger equilibrium following the parameter change.

Those results hold for multidimensional strategy spaces, be it discrete or continuous, functional spaces, as well as for payoffs which need not be smooth or concave. In order to obtain the desired results only the monotonicity properties of incremental payoffs and the order properties of strategies matter. The approach is based on monotone comparative statics results developed by Topkis (1978) and the application of Tarski's fixed point theorem to increasing functions (Tarski (1955)).

4 Bayesian games

The approach has proved useful when analyzing Bayesian games, in particular the difficult issue of existence of equilibrium in pure strategies with a continuum of types and/or actions. Results have been obtained for supermodular games with general action and type spaces (Vives (1990)); for games in which each player uses a strategy increasing in type in response to increasing strategies of rivals

(Athey (2001)); and for "monotone supermodular" games with general action and type spaces (Van Zandt and Vives (2007)). We will briefly describe the last contribution and apply it to the analysis of a global game example in the next section.

Let T_i (a subset of Euclidean space) be the set of possible types t_i of player i. The types of the players are drawn from a common prior distribution μ on $T = \prod_{i=0}^{n} T_i$, where T_0 is residual uncertainty not observed by any player. The action space of player i is a compact rectangle of Euclidean space A_i, and his payoff is given by the (measurable and bounded) function $\pi_i : A \times T \to \mathbb{R}$. The ex post payoff to player i when the vector of actions is $a = (a_1, ..., a_n)$ and when the realized types $t = (t_1, ..., t_n)$ is thus $\pi_i (a; t)$. Action spaces, payoff functions, type sets, and the prior distribution are common knowledge. The Bayesian game is described by $(A_i, T_i, \pi_i; i \in N)$.

A pure strategy for player i is a (measurable) function $\sigma_i : T_i \to A_i$ that assigns an action to every possible type of the player. Let Σ_i denote the strategy space of player i and identify two strategies if they are equal with probability 1. We can define a natural order in the strategy space $\Sigma_i : \sigma_i \leq \sigma'_i$ if $\sigma_i (t_i) \leq \sigma'_i (t_i)$, in the usual componentwise order, with probability one on T_i.

This formulation of a Bayesian game encompasses both common and private values as well as perfect or imperfect signals.

Van Zandt and Vives (2007) show the following result. Let $\Delta(T_{-i})$ be the set of probability distributions on T_{-i} and let player i's posteriors be given by the (measurable) function $p_i : T_i \to \Delta(T_{-i})$, consistent with the prior μ (the common prior assumption can be relaxed). Define a monotone supermodular game as follows.

1. Supermodularity in payoffs: π_i supermodular in a_i, and with increasing differences in (a_i, a_{-i}).

2. Complementarity between action and type: π_i has increasing differences in (a_i, t).

3. Monotone posteriors: $p_i : T_i \to \Delta(T_{-i})$ is increasing with respect to the partial order on $\Delta(T_{-i})$ of first–order stochastic dominance (a sufficient, but not necessary, condition is that the prior μ be affiliated).

In a monotone supermodular game there is a largest and a smallest Bayesian equilibrium and each one is in monotone strategies in type. There might be other equilibria that are in nonmonotone strategies but, if so, they will be between the largest and the smallest one, which are monotone in type. Furthermore, the extremal equilibria are increasing in the posteriors.

Monotone supermodular games have been applied to characterize equilibria in adoption games on graphs, multimarket oligopoly, team problems, and games of voluntary disclosure (see Vives (2005)).

5 Global games

Global games were introduced by Carlsson and van Damme (1993) as games of incomplete information with types determined by each player observing a noisy signal of the underlying state. The aim is to select an equilibrium with a perturbation of a complete information game. The authors show that in 2×2 games if each player observes a noisy signal of the true payoffs and if ex ante feasible payoffs include payoffs that make each action strictly dominant, then as noise becomes small an iterative strict dominance selects one equilibrium. The equilibrium selected is the risk-dominant one if there are two equilibria in the complete information game. Carlsson and van Damme do not explicitly consider supermodular games but when we have two equilibria in a complete information game, the game is one of strategic complementarities. Global games have been developed in a range of applications by Morris and Shin (2002). I will present here an example, a variation of the simple framework of Section 2 with incomplete information, to highlight the power of the supermodular game approach.

Suppose an agent must decide whether or not to adopt a new technology (or whether to "invest", "act", or "participate"). The action is $a_i = 0$ if there is no adoption and is $a_i = 1$ if there is adoption. The cost θ_i of adoption for agent i follows a normal distribution with mean μ_θ and variance σ_θ^2. The parameters θ_i and θ_j ($j \neq i$) are potentially correlated with covariance $\rho\sigma_\theta^2$ and correlation coefficient $\rho \in [0, 1)$. The benefit of adoption is $g(\widetilde{a})$, where $\widetilde{a}$ is the total mass adopting (between 0 and 1), and no adoption yields no benefit. We have thus a variation, with idiosyncratic random parameter, of the simple framework where the payoff to player i is:

$$\pi(a_i, \widetilde{a}; \theta_i) = a_i (g(\widetilde{a}) - \theta_i).$$

If $g' > 0$ then the game is monotone supermodular because $\pi(a_i, \widetilde{a}; \theta_i)$ has increasing differences in $(a_i, (\widetilde{a}, -\theta_i))$ (that is, $\partial^2\pi/\partial a_i \partial\widetilde{a} = g'(\widetilde{a}) > 0$ and $\partial^2\pi/\partial a_i \partial(-\theta_i) = 1 > 0$) and types are affiliated (because of normality). It follows that extremal equilibria exist, are symmetric (because the game is symmetric), and are in monotone (decreasing) strategies of the form $a_i = 1$ if and only if $\theta_i \leq \widehat{\theta}$. Let $g(\widetilde{a}) = \alpha\widetilde{a}$ with $\alpha > 0$ for purposes of illustration.

From the point of view of player i and given θ_i, the adopting mass when other players use the equilibrium threshold $\widehat{\theta}$ will be estimated by

$$\widetilde{a}(\theta_i) \equiv \Pr[\theta_j \leq \widehat{\theta} | \theta_i] = \Phi\left(\frac{\widehat{\theta} - (\rho\theta_i + (1-\rho)\mu_\theta)}{\sigma_\theta\sqrt{1-\rho^2}}\right),$$

where $\Phi(\cdot)$ is the cumulative distribution function of the standard normal. The agent will adopt if and only if $\alpha\widetilde{a}(\theta_i) \geq \theta_i$ and the best reply $r\left(\widehat{\theta}\right)$ is given implicitly by the solution in θ to

$\alpha\left(\Pr[\theta_j \leq \widehat{\theta}|\theta]\right) - \theta = 0$. The equilibrium threshold $\widehat{\theta}$ will satisfy

$$\hat{r}\left(\widehat{\theta}\right) = \widehat{\theta},$$

where

$$\hat{r}\left(\widehat{\theta}\right) \equiv \alpha\left(\Pr[\theta_j \leq \widehat{\theta}|\theta_i = \widehat{\theta}]\right) = \alpha\Phi\left(\sqrt{\frac{1-\rho}{1+\rho}}\left(\frac{\widehat{\theta}-\mu_\theta}{\sigma_\theta}\right)\right).$$

The solution will be unique if $\hat{r}' < 1$ or $\alpha\sqrt{(1-\rho)/(1+\rho)}\,(\sigma_\theta)^{-1}\phi(\cdot) < 1$, where $\phi(\cdot)$ is the density of the standard normal. It is then immediate[1] that the equilibrium will be unique when

$$\alpha\sqrt{\frac{1-\rho}{1+\rho}\cdot\frac{1}{2\pi\sigma_\theta^2}} < 1.$$

This will be so when the degree of strategic complementarity is not too strong. This may happen either because payoff complementarities are weak (α low); or because each player ex ante faces a large cost uncertainty (σ_θ high); or because the correlation of the costs is high (ρ close but not equal to 1). All three factors tend to lessen the strength of strategic complementarities.

Let $\alpha = 1$ in order to illustrate the effect of uncertainty. If costs are perfectly correlated then there are multiple equilibria for $\theta \in (0,1)$. In this case, there is complete information because a player, by knowing his own cost, knows the costs of any other player. However, a little bit of imperfect cost correlation (ρ close to 1) or a diffuse prior ($\sigma_\theta^2 \to \infty$) –following the idea of equilibrium selection in global games– will yield a unique equilibrium. Note, for example, that for $\alpha = 1$, $\hat{r}\left(\widehat{\theta}\right)$ tends to 1/2 either when $\sigma_\theta \to \infty$ or as $\rho \to 1$, yielding the unique solution $\widehat{\theta} = 1/2$. Figure 2 displays $\hat{r}(\cdot)$ for the case $\mu_\theta = 1/2$, and $\widehat{\theta} = \mu_\theta = 1/2$ is the equilibrium threshold (with the cases $\rho = .25$ and $\rho = .75$). In this case if $\sqrt{(1-\rho)/(1+\rho)}/\sqrt{2\pi\sigma_\theta^2} > 1$, two more equilibria appear.

Either with a diffuse prior or when the cost of a player gives very precise information about the costs of others, the (strategic) uncertainty of player i is maximal about the behavior of others. In both cases cases the player puts very little weight on prior information: when σ_θ^2 is very large because the prior is flat; when is ρ close to 1 because the type of the player predicts almost perfectly the types of others. This induces a best response for the player which is quite "flat", that is, not very sensitive to the threshold used by others and uniqueness obtains.

A standard procedure in a global game (e.g. Morris and Shin (2002)) would find the largest and smallest equilibrium thresholds by iterated elimination of dominated strategies, show that there is no loss of generality is assuming threshold strategies, and finally find conditions under which the largest and smallest equilibrium thresholds coincide.[2] By using the theory of (monotone) supermodular

[1] If $x \sim N(\mu, \sigma^2)$ then $f(\mu) = \left(\sigma\sqrt{2\pi}\right)^{-1}$, where f is the density of x.

[2] Frankel, Morris, and Pauzner (2003) obtain a generalization of the uniqueness result to games of strategic complementarities.

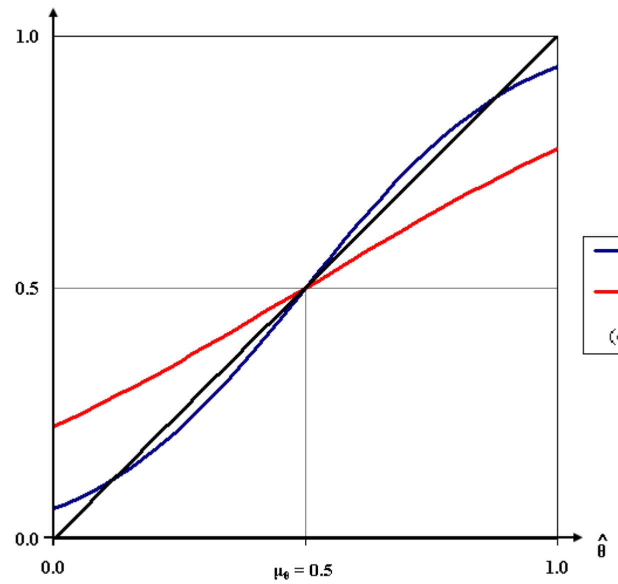

Figure 2

games we obviate the step of solving the iterated elimination of dominated strategies process, because in a supermodular game extremal equilibria are the outcome of iterated elimination of dominated strategies; we obtain immediately that extremal equilibrium strategies are of the threshold form –because in a monotone supermodular game extremal equilibria are monotone in type; and we bring forward the intuition for the uniqueness result in terms of lessening the strength of strategic complementarities. This helps to understand why in some occasions reducing noise and in others increasing it is necessary to obtain a unique equilibrium.

Indeed, we can start by noting that the game is monotone supermodular. This means that extremal equilibria exist and are in monotone (threshold) strategies. Those extremal equilibria can be found starting at extremal points of the strategy sets of players ($\widehat{\theta} = \infty$ or $\widehat{\theta} = -\infty$) and iterating using best responses. Typically, we must make sure that the process is not stuck at extremal points of the strategy space (and boundary assumptions may be used for this purpose). The extremal equilibrium thresholds bound the set of rationalizable strategies, and if the equilibrium is unique then the game is dominance solvable. The condition for equilibrium uniqueness is precisely that strategic complementarities are not too strong.

In the region where equilibrium is unique we can perform easily comparative statics analysis. For example, we can check that there is a multiplier effect of public information. An increase in μ_θ will have an effect on the equilibrium threshold $\widehat{\theta}$ over and above the direct impact on the best response of a player $\partial r/\partial \mu_\theta < 0$. Indeed, the prior mean μ_θ of θ can be understood as a public signal of precision $\left(\sigma_\theta^2\right)^{-1}$ and, exactly as in the simple framework,

$$\left|\frac{d\widehat{\theta}}{d\mu_\theta}\right| = \frac{|\partial r/\partial\mu_\theta|}{1-r'} > \left|\frac{\partial r}{\partial\mu_\theta}\right|$$

whenever the uniqueness condition ($r' < 1$) is met since the game is of strategic complementarities ($r' > 0$). The multiplier is largest when r' is close to 1, that is, when we approach the multiplicity of equilibria region. The multiplier effect of public information is emphasized by Morris and Shin (2002) in terms of the coordinating potential of public information beyond its strict information content. The reason is that public information becomes common knowledge and affects the equilibrium outcome. Every player knows that an increase in μ_θ will shift downward the best replies of the rest of the players and everyone will be more cautious in adopting.

The uniqueness property is nice in a game, but we can still perform comparative statics analysis in a game of strategic complementarities even if there are multiple equilibria. For example, in the uniqueness region, $r' < 1$, we have that $\frac{d\widehat{\theta}}{d\mu_\theta} = \frac{\partial r/\partial\mu_\theta}{1-r'} < 0$. In the multiple equilibrium region the result still holds for extremal equilibrium thresholds –or for reasonable out-of-equilibrium dynamics that eliminate the middle "unstable" equilibrium. Indeed, we know that extremal equilibria of monotone supermodular games are increasing in the posteriors of the players. A sufficient statistic for the posterior of a player under normality is $E(\theta_j|\theta_i) = \rho\theta_i + (1-\rho)\mu_\theta$, which is increasing in μ_θ for $\rho < 1$. It follows then that increasing μ_θ will increase the extremal equilibrium thresholds.

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