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# PREDICTING UTILITY UNDER SATIATION AND HABITUATION

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## Abstract

We introduce a modification of the discounted utility model that accounts for both habituation and satiation in intertemporal choice. Habituation level and satiation level are state variables that induce changes in preferences as those states vary. We examine several properties of our model, discuss willingness to pay for an additional unit of consumption, and characterize the optimal consumption path. Predicted utility under projection bias and narrow bracketing is compared to actual realized utility. We argue that projection bias and narrow bracketing successfully explain the hedonic treadmill in the research area of happiness and life satisfaction.

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# PREDICTING UTILITY UNDER SATIATION AND HABITUATION

## 1. Introduction

Present consumption influences the utility of future consumption in two important ways. First, it creates satiation, thereby reducing the satisfaction derived from future consumption in the near future. A rich meal for dinner seems less attractive after a heavy lunch. A trip to Hawaii may seem less attractive if one had traveled to Hawaii the previous year. Satiation wanes with the passage of time and has a half-life that depends on the nature of the good consumed. Second, present consumption contributes to habit formation and, therefore, increases future marginal utility. Classical music, outdoor activities and sports, sushi, and drugs are a few examples of habit-forming goods.

In discounted utility (DU) models, consumption independence is assumed whereby the utility of consumption in each period is computed afresh and is independent of past consumption. Baucells and Sarin (2006) present a modification of the DU Model that accounts for satiation (SA Model). Becker (1966); Pollak (1970), Ryder and Heal (1973), Wathieu (1997, 2004) propose models that modify the DU Model to account for habituation (HA Model).

Read, Lowenstein, and Rabin (1999) state that "The most important taste change effects are habit formation and satiation." In this paper, we propose a Habituation-Satiation (HS) Model that combines both habituation and satiation in the evaluation of consumption streams. In the HS Model, habituation is influenced by a parameter,  $\alpha$ , and satiation by a parameter,  $\gamma$ . The evaluation of a consumption stream and, hence, the optimal consumption plan, depends on the relative values of  $\alpha$  and  $\gamma$ ; when  $\gamma \rightarrow 0$ , the optimal consumption plan resembles that obtained using a habit formation model. Similarly, when  $\alpha \rightarrow 0$ , the optimal plan resembles that obtained using a satiation model. When both parameters tend to zero, then we converge towards a DU model.

In Section 2, we introduce the HS Model. In this model, the carrier of utility is the difference between consumption (x) and a habituation level or reference point (r). When x = r, a neutral utility level of zero is achieved. Consumption in excess of r induces positive experienced utility, but also contributes to the satiation level (y). In contrast, consumption below r produces a negative experienced utility and contributes to the reduction of y, and may produce craving, or a negative satiation level.

The satiation level (y) is an exponentially discounted sum of excess past consumption over the reference point. Similarly, the habituation level (r) is a weighted average (exponential smoothing) of past consumption. The experienced utility is v(x - r + y) - v(y). Here, v can be thought of as concave utility or an S-shaped value function as in prospect theory. In this paper, we take v to be the prospect theory value function in which zero is interpreted as a neutral state of neither satisfaction nor dissatisfaction. The overall utility is simply the discounted sum of experienced utilities in each period.

Habituation has been studied in economics for several decades. Satiation has been a cornerstone of economic analysis and is captured through diminishing marginal utility in a static analysis. Our combined habituation-satiation model permits predictions that are difficult to make through either model alone. Section 3 explores some properties of the HS Model. We show that the model satisfies the local substitution property and, therefore, a consumption of (1,1) or (2,0) would yield the same utility if the time interval between two periods is made small. This property simply says that a slice of cake now and another slice a few seconds later should yield about the same utility as two slices of cake now. The HA Model and the DU Model do not satisfy this property.

In Section 4, we examine the willingness to pay for a unit of consumption under various levels of state variables r and y. In a constant consumption case, we illustrate that the willingness to pay in the HS Model follows a richer pattern than what is possible in the DU, HA, or SA models.

In Section 5, we present the optimal consumption plan produced by the HS Model. In the pure habituation case ( $\alpha = 0, \gamma = 0$ ), the optimal consumption plan is increasing. In the pure satiation case ( $\alpha = 0, \gamma > 0$ ), the optimal plan has a U-shaped form (high consumption at the beginning and at the end and a smooth consumption path in the middle). Under general conditions, an increasing plan also emerges as optimal in the combined model.

In Section 6, we show that under projection bias (Loewenstein, O'Donoghue and Rabin, 2003), the predicted utility of current Consumption over time is lower than the actual realized utility. Projection bias leads an individual to believe that the future will be similar to the present (current r and y will change more slowly than they actually do). An increase in consumption does provide higher utility initially but one becomes accustomed to "the good life" and realizes less experienced utility than one had hoped for initially. On the flip side, Adam Smith's man with the wooden leg (Smith, 1759, Part III, Ch. 3) does not feel as miserable over time as one would predict at the beginning. Projection bias may also cause a prisoner to believe that incarceration was not as bad as he had anticipated. One implication of projection bias is that people buy more when they are hungry or consider a turkey meal for Christmas less appetizing after a Thanksgiving turkey dinner because they project their current satiation levels into the future.

In Section 7, we examine the optimal consumption pattern that maximizes peak utility. We show that craving is induced by abstaining from consumption which creates an unmet need ( $\gamma = 0$ ). Consumption then satisfies the need by creating a peak utility experience, as utility is obtained by using the steeper segment of the value function in the negative domain. This is observed when one enjoys lunch when hungry or a shower after a long hike. The phenomenon of craving is specific to the HS Model and cannot be captured by either the HS or the SA models.

In Section 8, we consider a discrete choice problem where in each period only one of the two goods can be consumed. We show that the optimal choice pattern provides both variety-

seeking and habituation. Of course, optimal planning can only be done if the effects of habituation and satiation are fully accounted for by the consumer.

While studying optimal planning under such a high level of sophistication is interesting, the more important insights from our model are obtained when certain biases prevent economic agents from properly accounting for the effects of current consumption on future utility. One such bias is narrow choice bracketing, recently discussed in Read, Loewenstein and Rabin (1999). Herrnstein and Prelec (1992) argue that people ignore negative internalities of current consumption on future utilities and thus make suboptimal choices. We show in Section 9 that under narrow choice bracketing, when each period decision is made independently, we obtain substantially inferior results. A varied choice, though locally less preferred, (e.g., vacation to a different destination, trying a new restaurant, extending the friendship circle), may maximize total experienced utility in the broad bracket evaluation.

In Section 10, we consider the discrete choice problem that allows for variable quantity. The optimal plan consists of an increasing consumption over time that alternates between two goods.

We agree with Read, Loewenstein and Rabin (1999) that satiation and habituation are important determinants of utility. Current consumption causes satiation in the short run and habituation in the long run, and through these factors impacts the experienced utility of future consumption. Useful insights into a wide range of human behavior such as craving, variety-seeking, and addiction can be obtained through our model. A rational person will simply anticipate satiation and habituation and, therefore, make optimal choices for lifetime consumption as well as for an idiosyncratic discrete choice situation. Narrow bracketing and projection bias, however, cause individuals to make sub-optimal choices with systematic and predictable results. We conclude in Section 11 with the implications of our analysis for consumer choice and rational decision-making.

## 2. A Combined Model of Habituation and Satiation

Let  $(x_1, x_2, ..., x_T)$  be a consumption stream. What is the total utility that a consumer obtains from such a stream? The Discounted Utility (DU) Model proposes that the total utility be evaluated as:

$$U_{DU}(x_1,...,x_T) = \sum_{t=1}^T \delta^{t-1} v(x_t)$$

where  $v(x_t)$  is the utility of consumption  $x_t$  in period t, and  $\delta^t$  is the discount factor associated with period t.

The DU Model was first proposed by Samuelson (1937) and subsequently axiomatized by Koopmans (1960) and Koopmans, Diamond and Williamson (1964) for countable infinite streams. The discount factor incorporates impatience, which is balanced by the desire to spread out consumption induced by the concavity of v. A key feature of the DU Model is the separability over time. Thus, the utility derived from present consumption is not affected by past consumption. Hence, it does not account for habituation or satiation.

Many goods are habit-forming, where present consumption increases the marginal utility of future consumption. People get accustomed to eating in higher-priced restaurants or staying in better-quality hotels as their income increases. This adaptation does not happen instantly, but over time their reference point relative to which utility is evaluated moves upward.

Several modifications of the DU Model have been proposed to incorporate the habituation phenomenon. Of these modifications, we adopt Wathieu (1997, 2004). Wathieu's model captures the key aspect of habituation (i.e., consumption today increases the marginal utility of future consumption) by introducing a reference point. Wathieu's finite-time model successfully explains the preference for increasing consumption paths. Reference points, denoted as habituation levels, are psychologically sound (Kahneman and Tversky, 1979; Rabin, 1998). In a reference-point model, zero utility can be interpreted as a neutral state of neither satisfaction nor dissatisfaction, and positive utility is obtained when the consumption level exceeds the habituation level. Specifically, Wathieu's habituation model (HA) would evaluate ( $x_1, x_2, ..., x_T$ ) as follows:

$$U_{HA}(x_1,...,x_T) = \sum_{t=1}^T \delta^{t-1} v(x_t - r_t),$$
  

$$r_{t+1} = \alpha x_t + (1 - \alpha) r_t, t = 1,..., T - 1, r_1 \text{ given}$$

Here,  $\alpha \in [0, 1]$  is the habituation factor, and  $r_t$  is the corresponding habituation level. When  $\alpha = 0$ , the HA Model reduces to the DU Model. When  $\alpha = 1$ , the reference point in the current period is simply the consumption in the last period. The habituation factor,  $\alpha$ , could be different for different goods. In a life-cycle consumption model, we assume a level of  $\alpha$  that reflects adaptation to the aggregate level of consumption in each period.

Besides habituation, a second important way in which consumption today affects future preferences is satiation. Baucells and Sarin (2006) introduce a satiation model in which the contribution of current consumption to experienced utility is through the satiation level achieved due to previous consumption. Thus, the carrier of utility is the increment from current satiation rather than current consumption. Suppose that the satiation level is y at the beginning of a period. A consumption x in this period yields a utility v(y + x) - v(y) rather than v(x). Clearly, when y = 0 and assuming v(0) = 0, the experienced utility is simply v(x). In this case, our model coincides with the DU Model, as the utility is computed afresh each period. Further, when x = 0, the experienced utility is also zero.

The total utility of  $(x_1, x_2, ..., x_T)$  in the Satiation Model is:

$$U_{SA}(x_1,...,x_T) = \sum_{t=1}^{T} \delta^{t-1} [v(y_t + x_t) - v(y_t)],$$
  
$$y_{t+1} = \gamma(y_t + x_t), t = 1, ..., T - 1, y_1 \text{ given}$$

where  $\gamma \in [0, 1]$  is the satiation retention factor, and  $y_t$  is the corresponding satiation level produced by previous consumption. It is easily seen that the satiation level is the cumulative discounted consumption in previous periods; that is,  $y_t = \sum_{s=1}^{t-1} \gamma^{t-s} x_s + \gamma^{t-1} y_t$ . For some consumptions (e.g., a tennis lesson), utility is derived over an extended period of time. Such goods should be modeled as durable goods with a consumption flow over time.

In the tennis lesson example, the utility of the first lesson is  $u(x_1)$ . If  $\eta$  is the learning decay factor, then the actual consumption in period 2 is  $\eta x_1 + x_2$ . In the DU Model, the total utility of  $(x_1, x_2)$  is:

$$U_{DU}(x_1, x_2) = u(x_1) + \delta u(\eta x_1 + x_2)$$

Thus, the sooner one takes the second tennis lesson, the higher the total utility will be because both  $\delta$  and  $\eta$  decay with time. We now apply the Satiation Model, keeping in mind that the actual consumption in period 2 is  $\eta x_1 + x_2$ . Hence,

$$U_{SA}(x_1, x_2) = u(x_1) + \delta[u(\gamma x_1 + \eta x_1 + x_2) - u(\gamma x_1)]$$

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Interestingly, the SA Model shows a tradeoff between bringing forward the second tennis lesson to profit from the learning achieved in the first lesson, and allowing some time to rest from the first lesson (satiation). This tension is realistic and would yield an optimal timing between lessons.

### Figure 1

Experienced Utility in the Combined Habituation-Satiation Model



Our next goal is to present a hybrid model, called the HS Model, for habituation and satiation. The combined model (Figure 1) is strongly reference-dependent: the carrier of utility is consumption in excess of the habituation level and the driver of satiation is also consumption in excess of the habituation level,  $r_t$ . In the HS Model, the utility of a neutral outcome (x = r) is zero. This is consistent with the Experienced Utility Model and the Prospect Theory Model (Kahneman, Wakker, and Sarin, 1997). Formally,

$$U_{HS}(x_1,...,x_T) = \sum_{t=1}^{T} \delta^{t-1} [v(y_t + x_t - r_t) - v(y_t)], \qquad (1)$$

$$r_{t+1} = \alpha x_t + (1 - \alpha)r_t, t = 1, ..., T - 1, r_1$$
 given, and (2)

$$y_{t+1} = \gamma(y_t + x_t - r_t), t = 1, ..., T - 1, y_1$$
 given. (3)

The combined model particularizes to the HA Model if  $\gamma = 0$ , to the SA Model if  $\alpha = 0$ , and to the DU Model if both  $\gamma$  and  $\alpha$  are zero. We will show that the HS Model explains some phenomena that cannot be explained by either the HA or the SA models alone. We recognize that, in some cases, either habituation or satiation is the main factor and that the full force of the HS Model may not be needed. The virtue of the combined model is that it enables the data to reveal the appropriate levels of the habituation and satiation parameters.

## 3. Properties of the Combined HS Model

A simple way to compare the experienced utility of the four model combinations is to consider a two-period problem. We fix consumption in the first period at  $x_1$  equal to one unit. If the initial satiation and habituation levels are zero, then the experienced utility in period 1 for all models is v(1), and the habituation and satiation levels for period 2 coincide with  $\alpha$  and  $\gamma$ , respectively. We now consider the experienced utility in period 2 derived from consuming  $x_2 = 0$ , 1, or 2 units. We assume that  $\alpha = \gamma = 0.5$ . Table 1 shows how the four different models yield different predictions.

Consider the consumption profile ( $x_1 = 1$ ,  $x_2 = 1$ ). As shown in the middle column of Table 1, the experienced utility in period 2 is the highest for the DU Model, as the utility is computed afresh.

### Table 1

Experienced Utility in Period 2 for Different Consumption Levels, Starting with  $r_1 = y_1 = 0$ , and after Having Consumed One Unit in Period 1. [ $\alpha = \gamma = 0.5$ ]

			$v(y_2+x_2-r_2)-v(y_2)$			
Model	$r_2 [= \alpha]$	$y_2[=\gamma]$	$x_{2} = 0$	$x_{2} = 1$	$x_{2} = 2$	
DU	0	0	v(0) = 0	v(1)	v(2)	
HA	0.5	0	v(-0.5)	v(0.5)	<i>v</i> (1.5)	
SA	0	0.5	v(0.5) - v(0.5) = 0	v(1.5) - v(0.5)	v(2.5) - v(0.5)	
HS	0.5	0.5	v(0) - v(0.5)	v(1) - v(0.5)	v(2) - v(0.5)	

## Figure 2

Direct and Indirect Effect of Consumption on Future Levels of Habituation and Satiation



In both the HA and the SA models, the experienced utility in period 2 is lower than that implied by the DU Model. This is because the higher reference point in the HA Model,  $r_2 = 0.5$ , and the higher satiation level in the SA Model,  $y_2 = 0.5$ , reduce utility. The utility in the HS Model is lower than in the SA Model. We now explore more formally the effects of current consumption on future habituation and satiation levels.

#### 3.1. The Effects of Current Consumption

To examine the effect that  $x_s$  has on utility in time t, t > s, we consider the effects that  $x_s$  has on future habituation and satiation levels. In Figure 2, one can visualize the dynamic effects of equations (2) and (3). According to these equations, both the satiation level and the habituation level are weighted sums of past consumption. The coefficient that  $x_1$  has in the expression for  $y_3$ , for instance, is the sum of the coefficients across all directed paths that connect  $x_1$  with  $y_3$ ; and the coefficient of a given path is the product of coefficients along this path. Thus,  $x_1$  appears in the expression for  $y_3$  multiplied by  $\gamma^2 - \alpha \gamma$ .

**Proposition 1** Both  $r_t$  and  $y_t$  are linear functions of  $x_{s,s} = 1, ..., t - 1$ . If t > s, then the coefficient of  $x_s$  in  $r_t$  and  $y_t$ , respectively, is given by:

$$\partial r_t / \partial x_s = \alpha (1 - \alpha)^{t-s-1},$$
(4)

$$\frac{\partial y_t}{\partial x_s} = [\gamma(1-\gamma)\gamma^{t-s-1} - \gamma\alpha(1-\alpha)^{t-s-1}]/(1-\alpha-\gamma) \text{ if } \alpha + \gamma \neq 1, \text{ and}$$
(5)

$$\partial y_t / \partial x_s = \gamma^{t-s-1} (\gamma - (1 - \gamma)(t-s-1)) \text{ otherwise.}$$
 (6)

**Proof.** Inspection of Figure 2 shows that  $x_1$  increases  $r_2$  in  $\alpha x_1$ , increases  $r_3$  in  $\alpha(1-\alpha)x_1$ , increases  $r_4$  in  $\alpha(1-\alpha)^2 x_1$ , etc., and (4) follows. Regarding equation (5), notice that  $x_s$  increases  $y_{s+1}$  in  $yx_s$ . The effect of  $x_s$  on  $y_{s+2}$  is a combination of a direct effect ( $\gamma^2$ ) and an indirect effect ( $-\alpha\gamma$ ) due to the mitigation effect that habituation has on satiation. The final effect of  $x_s$  on  $y_t$ , t > s, is given by:

$$\partial y_t / \partial x_s = \gamma^{t-s} - \alpha \gamma^{t-(s+1)} - \alpha (1-\alpha) \gamma^{t-(s+1)-1} \dots - \alpha (1-\alpha) \gamma^{t-s-2} \gamma$$
  
=  $\gamma^{t-s} - \alpha (1-\alpha)^{t-s-1} \sum_{q=1}^{t-s-1} \left[ \frac{\gamma}{1-\alpha} \right]^q$   
=  $\frac{\gamma (1-\gamma) \gamma^{t-s-1} - \gamma \alpha (1-\alpha)^{t-s-1}}{1-\alpha-\gamma}$ 

One can distinguish three effects of current consumption,  $x_s$ , on future utility. First, current consumption increases the future habituation level, as given by (4). The higher habituation levels produced by the first effect always reduce future experienced utility, but increase the marginal utility of future consumption. Second, current consumption increases the future satiation level via the positive coefficient  $\gamma^{t-s}(1-\gamma)$  in (5). Third, the interaction between satiation and habituation lowers future satiation via the negative coefficient  $\gamma \alpha (1 - \alpha)^{t-s-1}$ in (5). The net result of these two effects is always positive and equal to  $\gamma$  in the subsequent period (t = s + 1); however, if  $\alpha > 0$ , then  $\partial y_t / \partial x_s$  eventually becomes negative as t - sgrows larger. In fact, if  $\alpha > \gamma$  and  $t \ge s + 2$ , then  $\partial y_t / \partial x_s < 0$ . This interaction effect, which may produce  $\partial y_t / \partial x_s < 0$ , shows that *habituation mitigates satiation*. Because satiation is driven by consumption in excess of the habituation level, current consumption increases the future habituation level, which has the indirect effect of reducing the satiation level two or more periods ahead. Hence, current consumption increases satiation level in the next (and possibly some subsequent) period(s), but reduces the satiation level of the remaining periods. For those periods where  $\partial y_t / \partial x_s > 0$ , if v is concave, then the marginal utility of  $x_t$ unambiguously increases with  $x_s$ .

The fact that habituation mitigates satiation implies that, once habituated, a consumer is able to sustain large intakes without experiencing satiation. For instance, somebody used to classical music may listen to classical music for the entire day without experiencing satiation. On the contrary, a consumer just initiated to some consumption good for which he has little habituation may be easily satiated with relatively small amounts.

We expect  $\alpha$  and  $\gamma$  to depend on the time interval between periods. For satiation, as the time separation between periods increases, one expects larger decays in satiation levels. Hence,  $\gamma$  tends to 0 as the separation between time intervals increases (and to 1 if it decreases). With respect to the habituation level, one expects  $\alpha$  to increase (higher adaptation speed) over longer time intervals. Conversely, if the time interval shrinks, then we expect  $\alpha$  to tend to 0. In general, satiation is more pronounced for shorter time intervals and habituation occurs over longer time periods. Over time, one may adapt to higher-priced or quality restaurants, but, in short periods (e.g., daily), one may seek variety in the type of food consumed because of satiation. However, even for a large fixed time interval, there are goods for which  $\gamma$  could be large (e.g., movies, vacations), as well as goods for which  $\alpha$  could be small.

In *The Wealth of Nations*, Adam (Smith, 1776, p. 183) argues that "the desire of food is limited in every man by the narrow capacity of the human stomach, but desire of the conveniences and ornaments of buildings, dress, equipage, and household furniture, seems to have no limit or certain boundary." Food, shelter, clothing, sleep, and social relationships are goods for which  $\alpha$  is small; hence we call them *basic goods*. If the reference level stays relatively constant, then the evaluation of these goods is given by the SA Model. We expect that basic goods will produce satiation depending on the absolute intake value. As there is little habituation, basic goods produce high levels of experienced utility across their lifetime, assuming the satiation level is kept under control. Goods with a high  $\alpha$  are habituating goods, and the quantities one can consume of these goods can be quite large. The reference point for such goods, however, adapts to the level of consumption and the experienced utility reverts to the neutral level.

### 3.2. The Local Substitution Property

An important motivation for the form that we chose for the HS Model is the local substitution property. The property is compelling: consumption in the two-period stream (2, 0) should yield the same utility as the stream (1, 1) as the time separation between period 1 and 2 approaches zero. As argued in Baucells and Sarin (2006), neither the DU nor the HA models possess the local substitution property. This property says that eating two slices of cake now and no slices of cake a few seconds later should be equivalent to eating one slice of cake now and one slice of cake a few seconds later.

Let  $\Delta$  be the length of the time interval between periods. We now argue that if  $\Delta \rightarrow 0$ , then  $U_{HS}(x_1, x_2)$  and  $U_{HS}(x_1 + x_2, 0)$  both tend to  $v(x_1 + x_2)$ .

**Proposition 2** If  $\gamma > 0$ , then  $U_{HS}(x_1, x_2) \rightarrow U_{HS}(x_1 + x_2, 0)$  as  $\Delta \rightarrow 0$ .

**Proof of Proposition 2.** First, notice that as  $\Delta$  goes to zero, one should expect  $\alpha(\Delta)$  to go to zero, and  $\gamma(\Delta)$  to go to one, provided  $\gamma > 0$ . One natural way to keep the satiation half-life and the speed of adaptation approximately constant would be to set  $\gamma(\Delta) = \gamma^{\Delta}$  and  $\alpha(\Delta) = 1 - (1 - \alpha)^{\Delta}$ . We now show that both  $U_{HS}(x_1, x_2)$  and  $U_{HS}(x_1 + x_2, 0)$  tend to  $v(x_1 + x_2 - 2r_1 + y_1) - v(y_1)$ .

$$U_{HS}(x_1, x_2) = v(x_1 - r_1 + y_1) - v(y_1) + v(x_2 - (\alpha x_1 + (1 - \alpha) r_1) + \gamma(x_1 - r_1 + y_1)) - v(\gamma(x_1 - r_1 + y_1)) \rightarrow v(x_1 - r_1 + y_1) - v(y_1) + v(x_2 + x_1 - 2r_1 + y_1) - v(x_1 - r_1 + y_1) = v(x_1 + x_2 - 2r_1 + y_1) - v(y_1), and U_{HS}(x_1 + x_2, 0) = v(x_1 + x_2 - r_1 + y_1) - v(y_1) + v(-(\alpha(x_1 + x_2) + (1 - \alpha)r_1) + \gamma(x_1 + x_2 - r_1 + y_1)) - v(\gamma(x_1 + x_2 - r_1 + y_1)) \rightarrow v(x_1 + x_2 - r_1 + y_1) - v(y_1) + v(x_1 + x_2 - 2r_1 + y_1) - v(x_1 + x_2 - r_1 + y_1) = v(x_1 + x_2 - 2r_1 + y_1) - v(y_1).$$

A combined model in which satiation is driven by the absolute level of consumption, and not by consumption in excess of the habituation level, would not possess the local substitution property.

## 4. Willingness to Pay

In order to obtain a feel for how the model operates, we now consider how changes in the levels of satiation and habituation affect the willingness to pay for one unit of consumption. We define the willingness to pay (wtp) as the difference between the utility of consuming one unit and the utility of consuming zero units in a given period. Of course, willingness to pay depends on previous consumption via the current habituation and satiation levels y and r. Assuming a quasi-linear utility for money, the willingness to pay for one unit of consumption is:

$$wtp = [v(y+1-r) - v(y)] - [v(y-r) - v(y)] = v(1+y-r) - v(y-r)$$
(7)

Inspection of equation (7) reveals that *wtp* depends on y - r. For a concave *v*, *wtp* always decreases as y - r increases. For a prospect theory value function, *wtp* is non-monotonic because of diminishing sensitivity in both positive and negative domains. It is illustrative to consider values of *wtp* for different values of *r* and *y*. Table 2 performs this calculation. In our numerical examples throughout this paper, we employ the S-shaped power form:  $v(x) = x^{\beta}$ ,  $x \ge 0$ , and  $v(x) = -\lambda |x|^{\beta}$ , x < 0. We set the exponent to  $\beta = 0.5$ , and loss aversion parameter,  $\lambda = 2.25$ .

### Table 2

Willingness to Pay (= [v(y + 1 - r) - v(y)] - [v(y - r) - v(y)] for Different Levels of Habituation (column) and Satiation (row)

wtp	r=0	0.25	r=0.5	0.75	r=1	1.25	1.5
y=-1	2.25	1.39	1.16	1.03	0.93	0.86	0.80
-0.75	2.45	2.25	1.39	1.16	1.03	0.93	0.86
-0.5	2.30	2.45	2.25	1.39	1.16	1.03	0.93
-0.25	1.99	2.30	2.45	2.25	1.39	1.16	1.03
y=0	1	1.99	2.30	2.45	2.25	1.39	1.16
0.25	0.62	1	1.99	2.30	2.45	2.25	1.39
0.5	0.52	0.62	1	1.99	2.30	2.45	2.25
0.75	0.46	0.52	0.62	1	1.99	2.30	2.45
y=1	0.41	0.46	0.52	0.62	1	1.99	2.30
1.25	0.38	0.41	0.46	0.52	0.62	1	1.99
1.5	0.36	0.38	0.41	0.46	0.52	0.62	1

In Table 2, observe that for  $(y - r) \ge 0$ , the willingness to pay decreases as (y - r) increases. Thus, if satiation is above the reference point, greater satiation leads to a lower willingness to pay. Similarly, for  $(y - r) \le -1$ , the willingness to pay decreases as (y - r) decreases. For -1 < (y - r) < 0, the willingness to pay will either have a single peak or will increase monotonically as (y - r) decreases. The maximum willingness to pay is in the region where  $-1 \le (y - r) < 0$ . For food, it is easy to see that the desire for consumption, and therefore the *wtp*, is low when one is full. Similarly, in extreme hunger (e.g., fasting for a long period), one loses the desire for food. At a modest level of hunger, one most craves for food and has the highest desire to consume. Wathieu (2004) gives examples from marketing (e.g., consumer goods) in which the highest levels of *wtp* are observed at moderate habituation levels.

It is also interesting to observe how willingness to pay changes with time. The simplest analysis is to consider a constant consumption scheme  $x_t = 1$ , t = 1, 2, ... T. Figure 3 shows the results for four combinations of parameters that correspond to the four models. In the DU Model, *wtp* is constant and is equal to one. In the HA Model studied by Wathieu (2004), *wtp* is increasing. In the SA Model, *wtp* is decreasing. In the combined HS Model, *wtp* decreases first and then, once it starts increasing, continues to increase. In the early periods, habituation level is relatively low and, therefore, increasing satiation lowers *wtp*. Once habituation increases beyond a certain level, *wtp* also increases. Beginners who are not habituated to watching an opera or eating a specialty food (e.g., spicy or raw) are likely to pay less for these goods than when they become accustomed to them.

The highest level of positive experienced utility for a given value function can only be produced by the HS Model when y < 0. An interpretation of y < 0 is the accumulation of unmet need or craving. A satisfaction of the unmet need creates a high amount of pleasure (i.e., experienced utility). For example, assume y = -1 and r = 0. A consumption of x = 1 produces an experienced utility of v(1) in the HA Model, but a substantially large -v(-1) in the HS Model, especially if one further assumes loss aversion (see losses portion of Figure 4). Thus, to experience peak utility one must have some deprivation (e.g., a long hike in cold weather to appreciate a warm shower). Scitovsky (1976) argues that pleasure results from intermittent satisfaction of desires.

### Figure 3

Comparison of the Evolution of Willingness to Pay for Each Unit Along a Uniform Unit Consumption Path



In the HA Model, consumption below the reference point produces a high negative utility. For example, if r = 1, then x = 0 will produce a utility of v(-1). In the HS Model, the experienced utility may not be as negative if satiation is high. For example, set x = 0 so that the experienced utility is v(y - r) - v(y) < 0. If y is large and v is concave for gains, the utility reduction due to non-consumption occurs in a flat zone of the value function (see gains portion of Figure 4). This corresponds to the reaction "I don't mind skipping tennis today, because I have been playing all week".

Craving (y < 0) and the subsequent realization of high utility from consumption, or occasional abstinence from consumption without significant reduction in utility when the satiation level is high, are realistic features of the HS Model.

## 5. Optimal Consumption Path

Suppose that a decision maker wishes to optimally allocate an income (*I*) over consumption periods t = 1, ..., T. For simplicity, assume  $\delta = 1$  (no discounting), a constant unit price, and borrowing and saving at zero percent interest. The decision maker chooses ( $x_1, ..., x_T$ ) to solve the following optimization problem:

Max 
$$U(x_1,...,x_T) = \sum_{t=1}^{T} v(y_t + x_t - r_t) - v(y_t)$$
 (8)  
s.t.  $\sum_{t=1}^{T} x_t \le I$ ,

and  $r_t$  and  $y_t$  satisfying the updating equations (2) and (3).

#### Figure 4

Satisfying Unmet Needs Creates a Large Positive Utility (negative portion). Abstinence When Satiation is High Creates Small Disutility (positive portion)



The optimal consumption path given by the DU Model (with  $\delta = 1$  and  $\alpha = \gamma = 0$ ) is constant with  $x_t = I/T$ , t = 1, ..., T. In the HA Model ( $\gamma = 0$ ), the optimal consumption path is increasing for any sufficiently low initial reference point  $r_1$  (Loewenstein, O'Donoghue and Rabin, 2003, Lemma 1). In the SA Model ( $\alpha = 0$ ), the optimal path with no discounting is U-shaped: constant between period 2 and T- 1, and higher in periods 1 and T (Baucells and Sarin, 2006).

We now show that, under certain conditions on v, the optimal path in the HS Model is increasing with the possible exception of the first period. Further, we show that the satiation levels as well as the habituation levels are increasing.

Our key observation is that, given satiation levels  $y_1, y_2, ..., y_T, y_{T+1}$ , both the consumption levels and the habituation levels can be determined using (2) and (3). Indeed, noting that  $r_{t+1} - r_t = \alpha(x_t - r_t) = \alpha(y_{t+1}/\gamma - y_t)$  and  $x_t = r_t + y_{t+1}/\gamma - y_t$ , t = 1, ..., T, both  $r_t$  and  $x_t$  can be directly calculated as follows:

$$r_{t+1} = r_1 + \sum_{r=1}^{t} (r_{\tau+1} - r_{\tau}) = r_1 + \alpha \sum_{r=1}^{t} (y_{\tau+1}/\gamma - y_{\tau}), \text{ and}$$
(9)

$$x_{t} = r_{1} + (y_{t+1}/\gamma - y_{t}) + \alpha \sum_{r=1}^{t-1} (y_{\tau+1}/\gamma - y_{\tau})$$
(10)

As a consequence, the optimization program (8) can be re-formulated as one of choosing values  $(y_2, ..., y_T, y_{T+1})$  to find:

Max 
$$\sum_{t=1}^{T} v(y_{t+1}/\gamma) - v(y_t)$$
  
s.t.  $Tr_1 + \sum_{t=1}^{T} (1 + \alpha(T-t))(y_{t+1}/\gamma - y_t) \le I$ 

Here, the budget constraint is obtained by adding (10) across all periods. The first-order conditions are:

$$v'(y_t/\gamma) - \gamma v'(y_t) = \lambda[(1-\gamma)(1+\alpha(T-t)) + \alpha], t = 2, ..., T, and$$

$$v'(y_{t+1}/\gamma) = \lambda$$
(11)
(12)

If we define  $\hat{v}(y) \equiv \gamma [v(y/\gamma) - v(y)]$ , then we can rewrite (FOC) as:

$$\hat{v}'(y_t) = \lambda[(1 - \gamma)(1 + \alpha(T - t)) + \alpha], t = 2, ..., T$$
(13)

The solution is well-behaved if we assume that  $\hat{v}$  is strictly increasing and concave. As satiation in the HS Model is driven by utility increments, an increasing  $\hat{v}$  ensures that the marginal utility of consumption is positive for all levels of satiation. Concavity of  $\hat{v}$  ensures that the marginal utility of consumption is decreasing for all levels of satiation.

**Proposition 3** Let  $r_1 = y_1 = 0$ , and  $\delta = 1$ . If  $\hat{v}$  is increasing and concave, then (i)  $y_1 \le y_2 \le ... \le y_T \le y_{T+1}$ ; (ii) if  $\alpha \ge \gamma$ , then  $x_1 \le x_2 \le ... \le x_T$ ; and (iii) if  $\hat{v}''' \ge 0$ , then  $x_2 \le x_3 \le ... \le x_T$  for any  $\alpha$  and  $\gamma$ .

**Proof of Proposition 3**. Notice that the right-hand side of (13) is decreasing in *t*. Hence, if  $\hat{v}'$  is decreasing in *y* (i.e.,  $\hat{v}$  is concave), then the solution satisfies  $y_2 \le y_3 \le ... \le y_T$ . To see that  $y_T \le y_{T+1}$ , notice that:

$$\begin{aligned} \mathbf{v}'(y_T/\gamma) - \gamma \mathbf{v}'(y_T) &= \lambda(1 - \gamma + \alpha) = \lambda - \gamma \lambda(1 - \alpha/\gamma) \\ &= \mathbf{v}'(y_{T+1}/\gamma) - \gamma \mathbf{v}'(y_{T+1}/\gamma)(1 - \alpha/\gamma) \\ &\geq \mathbf{v}'(y_{T+1}/\gamma) - \gamma \mathbf{v}'(y_{T+1}/\gamma) \geq \mathbf{v}'(y_{T+1}/\gamma) - \mathbf{v}'(y_{T+1}/\gamma) \end{aligned}$$

If  $\alpha \ge \gamma$ , then  $x_t$ , t = 1, ..., T is necessarily increasing, as:

$$\begin{aligned} x_{t+1} - x_t &= \alpha(r_t - x_t) + (y_{t+2}/\gamma - y_{t+1}) - (y_{t+1}/\gamma - y_t) \\ &= \alpha(y_{t+1}/\gamma - y_{t+1}) + (y_{t+2}/\gamma - y_{t+1}) - (y_{t+1}/\gamma - y_t) \\ &= (y_{t+2} - y_{t+1})/\gamma + y_{t+1}(\alpha - \gamma)/\gamma + y_t(1 - \alpha) \ge 0 \end{aligned}$$

If  $\hat{v}'' \ge 0$ , then  $y_{t+1} - y_t \le y_{t+2} - y_{t+1}$ , t = 2, ..., T - 1, and

$$x_{t+1} - x_t = (y_{t+2} - y_{t+1})/\gamma - (y_{t+1} - y_t) + \alpha(y_{t+1}\gamma - y_t) \ge 0$$

If consumption levels are increasing, then so are the habituation levels.

#### 5.1. The Case of Power Utility

We now consider the optimal consumption path for the power utility function. The power form of the per-period utility has been widely used because of its mathematical tractability. Let  $v(x) = x^{\beta}$ , with  $0 < \beta < 1$ . One can check that in this case  $\hat{v}(y) = y^{\beta}(\gamma^{1-\beta} - \gamma)$  remains increasing and concave, with the same power exponent as the original *v*. In particular,  $\hat{v}'(y) = \beta(\gamma^{1-\beta} - \gamma)/y^{1-\beta}$  is a hyperbola, and (12) and (13) yield:

$$y_{t} = \left(\frac{\kappa_{t}}{\lambda}\right)^{1/(1-\beta)}, t = 2, ..., T, \text{ where } \kappa_{t} = \frac{\beta(\gamma^{1-\beta} - \gamma)}{(1-\gamma)[1+\alpha(T-t)+\alpha]}, \text{ and}$$
(14)  
$$y_{T+1} = \gamma \left(\frac{\beta}{\lambda}\right)^{1/(1-\beta)}$$
(15)

To find  $\lambda$ , we first re-write the budget constraint as:

$$I + (1 + \alpha(T-1)) y_1 - Tr_1 = y_{T+1}/\gamma + \sum_{t=2}^T \frac{(1-\gamma)(1+\alpha(T-t)) + \alpha}{\gamma} y_t$$
$$= \left(\frac{\beta}{\lambda}\right)^{1/(1-\beta)} + \sum_{t=2}^T \frac{(1-\gamma)(1+\alpha(T-t)) + \alpha}{\gamma} \left(\frac{\kappa}{\lambda}\right)^{1/(1-\beta)}$$

For tractability, assume that both the initial habituation and satiation levels are a fraction of the total consumption budget (i.e.,  $r_1 = \eta I$  and  $y_1 = \theta I$ ,  $0 \le \eta$ ,  $\theta$ , < 1). Of course, this includes, as a special case,  $r_1 = y_1 = 0$ . Under this assumption, it follows that  $1/\lambda^{1/(1-\beta)}$  is inversely proportional to *I*, as:

$$\lambda^{1/(1-\beta)} = \frac{\beta^{1/(1-\beta)} + \sum_{t=2}^{T} \left[ (1-\gamma)(1+\alpha(T-t)) + \alpha \right] \kappa_t^{1/(1-\beta)} / \gamma}{I(1+(1+\alpha(T-1))\theta - T\eta)}$$
(16)

Replacing (16) into (14) and (15) shows that the satiation levels are all proportional to *I*. But replacing the satiation levels in (9) and (10), together with  $r_1$  and  $y_1$ , yields that  $r_{t+1}$  and  $x_t$ , t = 1, ..., T, are also proportional to the budget.

For a given budget or income (*I*,) the optimal consumption planning problem discussed above provides the associated maximum total utility,  $U(I) = U(x_{1}^{*}, ..., x_{T}^{*})$ , where  $x_{t}^{*}$ , are optimal consumption levels. By varying *I*, one can repeatedly solve the planning problem and obtain the indirect utility of income. In the power case, experienced utility in each period is proportional to  $I^{\beta}$ , as is the indirect utility of income.

## 6. Projection Bias

In our HS Model, one must accurately forecast the impact of current consumption on the future reference point and the future satiation level to predict future experienced utility. There is considerable evidence in Loewenstein, Read, and Baumeister (2003) that people underestimate both adaptation and satiation. With a rising income, the reference point may shift upwards faster than predicted. Similarly, people buy too much when hungry or buy a three-day pass for museums and fail to account for the increasing levels of satiation.

Loewenstein, O'Donoghue, and Rabin (2003) provide evidence that, while people qualitatively understand that they will become adapted or satiated, they underestimate the magnitude of these changes. They project that future preferences will be more similar to current preferences. The bias is therefore known as projection bias.

### Figure 5

Predicted and Actual Utility of Constant Consumption of One Unit Above Initial Reference Point (diamonds) and One Unit Below Initial Reference Point (squares) [ $\alpha = \gamma = 0.2, \pi = 0.5$ ]



We first consider the simple cases of constant consumption of one unit above the initial reference point and of constant consumption of one unit below the initial reference point. We assume that the predicted reference and satiation levels are halfway between the current and actual levels. Figure 5 shows that under both cases, the experienced utility converges to the neutral level. People adapt to both positive and negative outcomes. Under projection bias, however, people predict higher levels of experienced utility for the desirable case and lower levels of experienced utility for the undesirable case. The impact of good fortune or misfortune is predicted to be more severe than it actually turns out to be (Brickman, Coates, and Janoff-Bullman, 1978).

#### 6.1. Buying on an Empty Stomach

It has been well documented that shoppers who are hungry over-buy food (Nisbett and Kanouse, 1968). Loewenstein, O'Donoghue, and Rabin (2003) argue that one reason for this sub-optimal behavior is projection bias, as the consumer projects the hungry state into the future. In our model, such a behavior is easily explained as the anticipated satiation level when hungry ( $y \le 0$ ) is lower than what it will turn out to be. In Table 3, it is shown that the purchase amount increases with the level of projection bias. The projection parameter,  $\pi$ , indicates that the predicted satiation level is  $\pi *$  current level +  $(1 - \pi) *$  actual level.

This table is constructed by modeling a shopper that, at the beginning of period 1, makes a purchasing decision for a 10-day period. The good under consideration has a satiation retention factor of  $\gamma^1 = 0.2$ . The consumer is supposed to decide how to divide the budget between this and another good, thus spending a total budget of I = 100. The other good does not exhibit satiation. When  $\pi = 0$ , there is no projection bias and the rational model predicts a lower purchase of the good that satiates (approximately 25% of good 1 and 75% of good 2). The higher the projection bias, the more of the satiating good one consumes. In the extreme, when  $\pi = 1$ , the consumer fully projects the initial satiation level  $y_1 = 0$  into the future and, hence, ignores satiation entirely. Therefore, the consumer splits equally the budget between the two goods.

### Table 3

The Effect of Projection Bias on Total Advanced Purchasing of a Satiation Good ( $X_1$ ) and a Non-Satiation Good ( $X_2$ ), and Effect on Actual and Predicted Utility [ $\alpha^1 = 0.1$ ,  $\gamma^1 = 0.2$ ;  $\alpha^2 = \gamma^2 = 0$ ; w = 1/2]

π	$X_1$	$X_2$	Actual Utility	Predicted Utility
0	25	75	18.2	18.2
0.25	29	71	18.2	18.7
0.5	33	67	18.1	19.3
0.75	39	61	17.7	20.2
1	50	50	17.0	22.4

Inducing over-purchase when consumers have a low current level of satiation has been used as a marketing tool. For instance, at the beginning of the ski season, when people are "hungry" for skiing, people predict that they may go skiing more often than they actually will. Hence, they may find an offer of 10 ski passes attractive. As the season unfolds and the satiation level for skiing increases, the consumer may end up not taking advantage of the package. Similarly, consumer may over-purchase vacation days on cruise ships or beach vacations, as one does not anticipate correctly the satiation associated with staying in the same place for several days.

## 7. Peak Utility

In some cases, the focus may be on peak utility rather than total utility. Varey and Kahneman (1992) show that in retrospective evaluation (e.g., how good a vacation was) the peak and end experiences play a dominant role. Hence, the decision maker may want to maximize the peak utility of such an extended experience. Consider the problem of determining an optimal consumption plan that maximizes peak utility subject to the budget constraint:

$$\begin{aligned} \operatorname{Max}_{(x_1,\ldots,x_T)} & \operatorname{Max}_t \left\{ v(y_t + x_t - r_t) - v(y_t) \right\} \\ \text{s.t.} & \sum_{t=1}^T x_t \leq I, \end{aligned}$$

and  $r_t$  and  $y_t$  satisfy the updating equations (2) and (3).

In the DU, HA, and SA models, peak utility is maximized by consuming *I* in the last period. In the HS Model, however, consumption of a small amount in an early period increases the reference point; a few periods of no consumption then creates craving (y < 0), and, finally, a large consumption in the final period gives the peak utility. In our example with I = 100,  $\alpha = 0.3$ , and  $\gamma = 0.5$ , the peak utility is maximized by consuming 74 units in the final period, and consuming 26 units 5 periods before. In the remaining periods, consumption is zero. The realized peak utility is 12 compared to 10 if all 100 units are consumed in the last period. Promotional tools, such as a brief stay in a luxury resort, are designed to create desire.

Maximization of peak utility may not be rational in prospective planning. Kahneman, Wakker and Sarin (1997) distinguish between the desired goal of maximizing total utility and retrospective evaluation for which peak utility plays a major part. Our model suggests that voluntary abstinence, which creates heightened need, produces the peak experienced utility from subsequent consumption. For example, an outing to celebrate an anniversary is made more enjoyable if one follows a simple and routine life for a few days before the big event.

## 8. Discrete Choice Problem

In this section we consider a problem in which a consumer chooses one of two goods in each time period. The good quantity cannot be modulated and is set to one. An example of such a choice is having Indian food or American food for dinner on a given evening. If both goods are DU goods (i.e., produce no habituation and no satiation), then the choice problem is trivial. As utility does not change from one period to the next, it is optimal to choose the same good each period. Under satiation and habituation, however, the choice of a good now impacts its future experienced utility and, therefore, future choice. A rational consumer will choose a sequence that maximizes total utility. Assume that experienced utility in each period is additively separable. The consumer chooses a bundle by solving:

$$\begin{aligned} &\text{Max } \sum_{t=1}^{T} w \left[ v \left( y_{t}^{a} + x_{t}^{a} - r_{t}^{a} \right) - v \left( y_{t}^{a} \right) \right] + (1 - w) \left[ v \left( y_{t}^{b} + x_{t}^{b} - r_{t}^{b} \right) - v \left( y_{t}^{b} \right) \right] \end{aligned} \tag{17}$$
  
s.t.  $x_{t}^{a}, x_{t}^{b} = 0 \text{ or } 1, t = 1, ..., T$   
 $x_{t}^{a}, x_{t}^{b} = 1, t = 1, ..., T$   
 $r_{t+1}^{a} = \alpha^{a} x_{t}^{a} + (1 - \alpha^{a}) r_{t}^{a}, t = 1, ..., T - 1, r_{1}^{a} \text{ given},$   
 $r_{t+1}^{b} = \alpha^{b} x_{t}^{b} + (1 - \alpha^{b}) r_{t}^{b}, t = 1, ..., T - 1, r_{1}^{b} \text{ given},$   
 $y_{t+1}^{a} = \gamma^{a} \left( y_{t}^{a} + x_{t}^{a} - r_{t}^{a} \right), t = 1, ..., T - 1, y_{1}^{a} \text{ given, and}$   
 $y_{t+1}^{b} = \gamma^{b} \left( y_{t}^{b} + x_{t}^{b} - r_{t}^{b} \right), t = 1, ..., T - 1, y_{1}^{b} \text{ given.}$ 

Essentially, there are  $2^T$  possible sequences and the consumer chooses the sequence that produces the highest sum of experienced utility over *T* periods. To gain insight, we use an example in which T = 10 and the second good is assumed to be a DU good with  $\alpha^b = \gamma^b = 0$ . In Table 4, we show the optimal sequence for the HA, SA, and HS models. In Table 4, *A* and *B* denote ( $x_t^a = 1, x_t^b = 0$ ) and ( $x_t^a = 0, x_t^b = 1$ ), respectively.

For the HA Model, the optimal sequence has the increasing path property; that is, if it is optimal to consume *A* in period *s*, then it is optimal to continue consuming *A* in periods t > s. Hence, a consumer may consume *A* throughout, or begin with *B* and switch to *A* permanently at some intermediate period. A segment of Starbucks coffee consumers exhibit this loyalty.

#### Tabla 4

Optimal Discrete Choice between A and B [w = 2/3;  $\alpha^b = \gamma^b = 0$ ]

	HA Model	SA Model	HS Model
	$\alpha^{a} = 0.05$	$\alpha^{\rm a} = 0$	$\alpha^{\rm a}=0.05$
Period	$\gamma^{\mathrm{a}}=0$	$\gamma^{\mathrm{a}}=0.2$	$\gamma^a=0.2$
1	А	А	А
2	А	В	В
3	А	А	А
4	А	В	В
5	А	А	А
6	А	В	В
7	А	А	А
8	А	В	А
9	А	А	А
10	А	А	А

For the SA Model, one observes *variety-seeking* behavior. The consumer switches from A to B and back to A at regular intervals. The consumption of B takes as many periods as necessary to bring the satiation level of A below some threshold, after which consuming A is again optimal. Clearly, the SA Model also permits the trivial case when consuming only one of the two goods is optimal all the time.

For the HS Model, both habituation and satiation effects are involved. Thus, the optimal consumption path is more complex. The consumer may start by switching often between A and B due to variety seeking behavior induced by satiation of A. As the consumer develops habituation for A, he will choose B infrequently as time passes. At some point, the consumer may decide to permanently switch to A. An example may be switching between playing golf and tennis, playing golf with increasing frequency until ultimately switching solely to

golf. More broadly, casual observation indicates that young people seek variety in their activities, whereas older people tend to settle on a few, very established habits.

The optimal plan in any of the HA, SA, or HS models depends on the parameters  $\alpha$  and  $\gamma$ . It is important to note, however, that the pattern of variety seeking in early periods, less frequent switching in intermediate periods, and finally a permanent switch to one of the goods is consistent only with the HS Model. Thus, the HS Model does not simply include the HA, SA, and DU models as special cases; it permits a richer behavior than none of the other three models permits under any set of parameter values.

There is, however, a delicate balance between seeking variety and starting a new habit. For goods with low habituation (small  $\alpha$ ), variety mitigates the effects of satiation. But highly habituating goods induce negative utility when not consumed. The total utility may therefore be maximized by deliberately choosing only a few habituating goods. Parents usually try to keep children from acquiring time-consuming or expensive habits.

In an example with three symmetric goods, each with  $\alpha = 0.05$  and  $\gamma = 0.2$ , and equally weighted additive separable utilities, we find that the optimal sequence is to pick any two of these three goods and to exclude the third, and then to alternate consumption between the two chosen goods. Thus, (*A*, *B*, *A*, *B*,...) or (*A*, *C*, *A*, *C*,...) are both optimal sequences. Therefore, starting with homogenous preferences for a population, one may observe heterogeneous outcomes, e.g., people choosing and liking different consumption goods based on some arbitrary initial selection. Examples include type of cuisine, favorite sport team, and hobbies.

In the above example, if  $\alpha$  is set to 0.01 or less, then the optimal sequence, (*A*, *B*, *C*, *A*, *B*, *C*,...), alternates between the three goods. Thus, when habituation is low and satiation is high, more variety is indeed better. At the other extreme, if  $\alpha$  is equal to 0.1 or more, then there are three equivalent optimal sequences, which consist of picking the same good in each period, say (*B*, *B*, *B*,...), which avoids initiating a habit in the other two goods.

# 9. Narrow Bracketing

Optimal planning over the entire horizon (*T*) requires considerable cognitive effort as there are significant interactions across periods. People often use local optimization rather than global optimization to simplify the choice process. In a traveling salesman problem, for example, a heuristic to visit the closest unvisited city reduces the computational burden involved in a global optimization. Read, Loewenstein, and Rabin (1999) provide evidence that people often narrowly bracket the choices and thus focus on the local consequences of their choices. Herrnstein and Prelec (1992) argue that people ignore or under-appreciate interactions across periods. In our example, one form of narrow bracketing is simply to sequentially choose the highest utility alternative *A* or *B* in each period. In each period, one observes  $y_t$  and  $r_t$  (updated satiation and reference levels) and chooses the alternative (*A* or *B*) that yields the highest utility in that period alone.

In Table 5, an example illustrates that under narrow bracketing A is preferred to B in each of the ten periods. In this example, the utility of A is decreasing over time, but is still greater than the utility of B in each period. Without narrow bracketing, the optimal choice considers all ten periods together; in period 2, B is chosen despite its lower utility (0.29 versus 0.42). This is because the utility of A in period 3 jumps to 0.54, instead of 0.39 had locally optimal A been chosen in period 2, illustrating that the optimal choice takes into account the impact of earlier

choices on the utilities associated with later choices. To some degree, consumers do recognize such internalities: "let's eat a light lunch since we are going to a fancy restaurant for dinner." In many situations, however, choices are presented and made separately in each period and consumers may not take a holistic perspective. Simonson (1990) and Simonson and Winer (1992) have found that in a sequential choice condition, people tend to choose the same item in each period. For a simultaneous choice condition, people use a broad perspective and favor variety. For a simultaneous choice, the interactions are made more transparent (e.g., reservations in advance at restaurants for each day of the three days of a vacation) and, therefore, a consumer is more likely to take into account these interactions. For a sequential choice (e.g., making a reservation at a restaurant for just the same day), one does account for adaptation and satiation from past consumption, but may fail to see the impact of current choice on future utility.

Narrow bracket choices can sometimes be substantially inferior to the globally optimal choice. A middle ground, however, is to use a heuristic or a decision rule (tennis on Wednesday evening and golf on Saturday, go to a party only on one weekend evening, limit TV watching during weekdays) that to some degree takes into account preference interactions across periods. In Table 5, a heuristic of alternating between *A* and *B* gives a reasonable solution.

Both habituation and satiation impact future experienced utility. A rational person will properly account for these interactions and choose the sequence of consumptions that maximizes global utility. Narrow bracketing may, however, lead to over-consumption of addictive goods as their deleterious long-run effects may not be apparent on a daily basis until it is too late. Similarly, one may over-consume attractive, but highly satiating, goods (Herrnstein and Prelec, 1992).

	Narrow Bracket	Broad Bracket	Broad Bracket
Period	Sub-Optimal Choice	Optimal Choice	Heuristic Choice
1	A [0.67]	A [0.67]	A [0.67]
2	A [0.42]	B [0.29]	B [0.29]
3	A [0.39]	A [0.54]	A [0.54]
4	A [0.38]	B [0.25]	B [0.25]
5	A [0.37]	A [0.55]	A [0.55]
6	A [0.36]	B [0.19]	B [0.19]
7	A [0.35]	A [0.56]	A [0.56]
8	A [0.34]	A [0.39]	B [0.09]
9	A [0.33]	A [0.36]	A [0.59]
10	A [0.32]	A [0.35]	A [0.38]
Total Utility	Total Utility [3.91]		[4.12]
Rank	$88^{th}$ best	$1^{st}$ best	$4^{th}$ best

### Table 5

Narrow Bracket and Optimal Choice [ $\alpha = 0.05$ ;  $\gamma = 0.2$ ; w = 2/3]

# 10. Discrete Choice with Variable Quantity

We now consider the problem of allocating a budget (I) to two goods, A and B, over T periods. In each period, only one of the two goods can be consumed, either A or B. Essentially, this is the same as the discrete choice problem, but now both the type of good and its consumption quantity are optimally determined.

$$\begin{aligned} &\text{Max } \sum_{t=1}^{T} w \left[ v \left( y_{t}^{a} + x_{t}^{a} - r_{t}^{a} \right) - v \left( y_{t}^{a} \right) \right] + (1 - w) \left[ v \left( y_{t}^{b} + x_{t}^{b} - r_{t}^{b} \right) - v \left( y_{t}^{b} \right) \right] \end{aligned} (18) \\ &\text{s.t.} \quad \sum_{t=1}^{T} x_{t}^{a} + x_{t}^{b} \leq I, \\ &x_{t}^{a} x_{t}^{b} = 0, \end{aligned}$$

together with the four updating equations for  $r^a$ ,  $r^b$ ,  $y^a$ , and  $y^b$ , as in (17). In Table 6, the results of a numerical example are presented. In this example, goods A and B are symmetric in the sense that both have the same parameters of habituation and satiation, and w = 1/2. We observe that, because of habituation, the budget allocated is increasing over time; but because of satiation, consumption is alternating between Good A and Good B. An example of such a pattern may be that an individual's budget for meals increases over time, but the individual may alternate between the type of food or restaurant. This is consistent with the observation of Lyubomirsky, Sheldon, and Schkade (2005) that the pernicious effects of habituation can be attenuated by attending to the timing and variety of consumption. Scitovsky (1976) similarly argues that pleasure can be enhanced through novelty and variety.

### Table 6

Optimal Consumption Path in a Discrete Choice with Variable Quantity  $[\alpha^a = \alpha^b = 0.05; \gamma^a = \gamma^b = 0.2; w = 1/2]$ 

Period	Choice	Quantity A	Quantity B	Budget Allocation
1	В	-	3.84	3.84
2	А	4.80	-	4.80
3	В	-	5.27	5.27
4	А	6.48	-	6.48
5	В	-	7.20	7.20
6	А	9.52	-	9.52
7	В	-	10.70	10.70
8	А	13.50	-	13.50
9	В	-	19.30	19.30
10	А	19.30	-	19.30

## 11. Conclusions

In this paper, we have proposed a model that incorporates both habituation and satiation into evaluating time streams of consumption. Present consumption creates satiation, but also contributes to habit formation, thereby influencing the utility of future consumption. In our model, habituation is influenced by a parameter ( $\alpha$ ) and satiation by a parameter ( $\gamma$ ); thus, the evaluation of a consumption stream and the optimal consumption plan depends on the relative values of  $\alpha$  and  $\gamma$ .

The speed of adaptation ( $\alpha$ ) could be sign-dependent. For certain goods, one could imagine having  $\alpha^+ > \alpha^-$  (i.e., habituation levels adapt to current consumption faster when consumption exceeds the reference level and diminish slowly when consumption is below the habituation level). Similarly,  $\gamma$  could be different, as satiation increases at a faster rate with consumption than the rate at which it diminishes with no consumption.

A prediction of our model in discrete choice situations is that a consumer will seek variety in the early periods to mitigate the effects of satiation, but will gradually shift consumption towards highly habituating goods. A rational consumer will limit the amount of habitforming goods consumed by not initiating consumption of some goods. Consumers may start with identical preferences, but because of some arbitrary initial choice may gravitate towards different choices. For example, people initially may choose a sport activity such as golf, tennis, or running on some social or circumstantial basis, but then become habituated and continue to indulge in the same sport.

Perhaps the more important insights from our model are obtained when certain biases prevent consumers from properly accounting for the effects of current consumption on future utility. We consider narrow choice bracketing and projection bias and study how these biases impact the chosen consumption plan. These biases produce a gap between predicted and actual, realized utility.

In narrow choice bracketing, a consumer considers one period at a time and uses the optimal allocation for that period alone. This myopic optimization produces significantly inferior results if the interaction across periods through habituation and satiation is significant. It may, for example, lead to choosing the same good in each period even though a varied choice, though locally less preferred, may maximize total utility in the broad bracket evaluation.

A person with projection bias believes that the future will be similar to the present (i.e., the current reference point and satiation level will change more slowly than they actually do). Such a person will predict high experienced utility for constant consumption over the initial reference point, but will actually realize a lower experienced utility. Thus, one thinks that more money will buy more happiness when in fact it does not (Baucells and Sarin, 2007). One, however, need not accept the neutral utility as unavoidable. A greater allocation to basic goods and a careful selection of few habit-forming goods may improve experienced utility. A person may become adapted to a fancy car or staying in nice hotels, but always enjoys a nice meal and conversations with friends.

We suggest a few directions for future research. The first is to examine the implications of our model in the context of time horizon uncertainty. It is likely that the optimal consumption path will not be as increasing as we have obtained. Second, it will be interesting to explore the full implications of our model to well-being and happiness research. We have provided here only the most transparent result under projection bias. Third, an axiomatic study of our model is worth exploring. Finally, through effort, one may reduce projection bias or moderate adaptation speed; a comprehensive model of time allocation and consumption path determination may provide further insights into how people may counter the pernicious effects of habituation and satiation.

While the habituation-satiation model we propose is more complex than the DU or the HA models, we have shown that the added flexibility it brings, is needed to account for well-documented preferences and behaviors. Heaton (1995, p. 681) finds "evidence for a preference structure in which consumption at nearby dates is substitutable and where habit over consumption develops slowly." Our model shows that sophisticated patterns of variety seeking, selective habituation, and increasing consumption path are consistent with the rational choice of a consumer who experiences habituation and satiation.

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