IESE Business School
University of Navarra
SP-SP

Working Paper
WP-854
April, 2010

COLLUSIVE NETWORKS IN MARKET-SHARING AGREEMENTS UNDER THE PRESENCE OF AN ANTITRUST AUTHORITY

Flavia Roldán

IESE Business School – University of Navarra
Av. Pearson, 21 – 08034 Barcelona, Spain. Phone: (+34) 93 253 42 00 Fax: (+34) 93 253 43 43
Camino del Cerro del Águila, 3 (Ctra. de Castilla, km 5,180) – 28023 Madrid, Spain. Phone: (+34) 91 357 08 09 Fax: (+34) 91 357 29 13

The Public-Private Sector Research Center is a Research Center based at IESE Business School. Its mission is to develop research that analyses the relationships between the private and public sectors primarily in the following areas: regulation and competition, innovation, regional economy and industrial politics and health economics.

Research results are disseminated through publications, conferences and colloquia. These activities are aimed to foster cooperation between the private sector and public administrations, as well as the exchange of ideas and initiatives.

The sponsors of the SP-SP Center are the following:

- Accenture
- Ajuntament de Barcelona
- Caixa Manresa
- Cambra Oficial de Comerç, Indústria i Navegació de Barcelona
- Consell de l'Audiovisual de Catalunya
- Departament d' Economia i Finances de la Generalitat de Catalunya
- Departament d' Innovació, Universitats i Empresa de la Generalitat de Catalunya
- Diputació de Barcelona
- Endesa
- Fundació AGBAR
- Garrigues
- Mediapro
- Microsoft
- Sanofi Aventis
- ATM, FGC y TMB

The contents of this publication reflect the conclusions and findings of the individual authors, and not the opinions of the Center's sponsors.

COLLUSIVE NETWORKS IN MARKET-SHARING AGREEMENTS UNDER THE PRESENCE OF AN ANTITRUST AUTHORITY*

Flavia Roldán[1]

Abstract

This article studies how the presence of an antitrust authority affects market-sharing agreements made by firms. These agreements prevent firms from entering each other's market. The set of these agreements defines a collusive network, which is pursued by antitrust authorities. This article shows that while in the absence of the antitrust authority, a network is stable if its alliances are large enough when considering the antitrust authority, and more competitive structures can be sustained through bilateral agreements. Antitrust laws may have a pro-competitive effect, as they give firms in large alliances more incentives to cut their agreements at once.

JEL Classification Numbers: D43, K21, L41

Keywords: market-sharing, economic networks, antitrust authority, oligopoly

[1] Public-Private Sector Research Center, IESE

*This article is a revised version of Chapter 1 of my Ph.D. thesis submitted to the Universidad Carlos III de Madrid. I am grateful to Antonio Cabrales and Luis Ubeda for their advice, guidance and support. I have greatly benefited from comments and useful suggestions from Natalia Fabra, Carlos Ponce, Antonio Romero, José Penalva, Guillermo Caruana, and Juanjo Ganuza. I thank seminar participants at the Student Workshop at the Universidad Carlos III, the 2007 EARIE Meetings (Valence, Spain), XXXII Simposio de Análisis Económico, BBVA (January 2008), the University of Vienna (February 2008), IESE Business School (March 2008), Universitat de les Illes Balears (March 2008), CRESSE Conference, Chania (2009), ESEM 2009 Annual Meeting in Barcelona, Spain Jornadas de Economía Industrial (September 2009), Vigo, Spain. The usual disclaimers apply.

†E- mail address: froldan@iese.edu

1 Introduction

Reciprocal market-sharing agreements between firms are agreements by which firms divide up a market and agree not to enter each other's territory. These agreements are an anti-competitive practice; moreover, if after an investigation, the antitrust authority finds proof of market-sharing agreements, the firms involved are penalized.

The goal of the present article is to study how the presence of an antitrust authority affects the market-sharing agreements made by firms. We examine the network structure that arises when each firm takes into account the cost, imposed by competition authorities, from signing these collusive agreements.

Antitrust authorities spend a substantial amount of time and effort attempting to deter collusive market-sharing agreements. An example that stresses the importance of the problem studied and also helps us to understand the problem itself is the following. In October 2007, the Spanish Competition Authority (Comisión Nacional de la Competencia) fined the savings banks BBK, Kutxa, Caja Vital and Caja Navarra over €24 million.[1] Between 1990 and 2005, the cartel's members had agreed to carve up markets. In a the minutes of a meeting held by the members of the cartel on February 1990,[2] we it was noted that the top representatives of the "Basque Savings Banks and also Navarra Savings Bank have reaffirmed their commitment to maintain the territorial status quo ... thus avoiding competition among themselves and [they] agreed that the framework of the Federation remains the forum for information and sharing decisions on expansion, ... in the traditional way of opening new offices..."

Accordingly, none of the savings banks in the cartel opened any branch in each other's "traditional" territory (while conducting a remarkable territorial expansion in other provinces, especially near the borders).

Consequently, this kind of agreement reduces competition in a market and thus they damages to consumers.

In this article, market-sharing agreements are modeled as bilateral agreements, whereby firms commit to staying out of each other's market. The set of these reciprocal agreements gives rise to a collusive network among firms.

We chose a network framework because the structure of the relationship is important. Let us consider an antitrust authority defined by the probability of inspection and by a fine that is imposed on firms that are proved guilty of market-sharing agreements. Assume that when the antitrust authority inspects, it is able to detect, without error, the existence of a collusive practice. Let us consider Figure 1.

---

[1] It is the second-largest fine that has been imposed by the Spanish Competition Authority.

[2] See Expediente 617/06. Comisión Nacional de la Competencia, Spain (minutes 02-06-90). http://www.cncompetencia.es/Inicio/Expedientes/tabid/116/Default.aspx?numero=617/06&ambito=Conductas

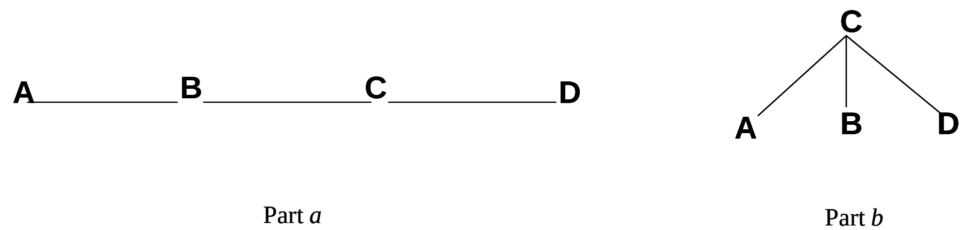

Figure 1.

Here, A,B,C and D represent four savings banks and the lines between them represent the existence of a market-sharing agreement between firms. Therefore, for example, savings bank C is linked by a market-sharing agreement with savings banks B and D.

Each part of the figure depicts a different structure of relationships among savings banks. In part *a*, each firm is connected to others as in a line. Furthermore, in part *b*, all savings banks are connected with savings bank C but are not linked to each other. In this case, they form a star.

In part *a* of the figure, if the antitrust authority inspects savings bank C, the antitrust authority may only destroy the market-sharing agreements that C holds with savings banks B and D. In part *b* of the figure, however, when the antitrust authority inspects C, it could destroy the *entire* network of relationships. In such a case, it is able to detect the agreements that savings bank C holds with savings banks A, B and D.

Therefore, the antitrust authority is more successful in the second case than in the first case. If the antitrust authority knows what the structure of relationships among the firms is, then it may concentrate its efforts in order to pick up the firm in the central position, as by doing so, it is able to destroy the entire network of relationships. Consequently, the structure of the relationships is important, which both the firms the competition authority should take into consideration when defining their actions.

The current article answers the question of how the structure of collusive networks interacts with the antitrust policy that tries to deter such collusive practice and which are their implications on the competition.

We first study the actual probability of being discovered in the collusive network framework. We show that the probability of being caught depends on the agreements that each firm has signed. That is, the probability of firm i's being detected depends not only on whether firm i is inspected by the antitrust authority but also on whether any firm that has formed an agreement with i is inspected. Therefore, if a firm is inspected and a market-sharing agreement exists, then it is detected, and the firms involved are penalized. However, the firm in consideration may be detected without being inspected because any firm that has an agreement with it is inspected.

We then provide a characterization of a stable network. While in the absence of the antitrust authority, a network is stable if its collusive alliances are large enough, when the antitrust authority is considered, more structures that are competitive can be sustained through bilateral agreements.

We also find a threshold that determines the minimum effort that the competition authority should exert in order to completely deter the existence of local monopolies in all markets.

Furthermore, when the notion of strong stability is considered, the antitrust authority has a pro-competitive impact. That is, as the probability of inspection increases, firms in large alliances have more incentives to renege on all their agreements at once, which might lead to a breakdown of collusion.

This article brings together elements from the literature of collusion (particularly, market-sharing agreements), networks, and law enforcement.

Networks is currently a very active field of research. Prominent contributions to this literature include, among others, Jackson and Wolinsky (1996), Goyal (1993), Dutta and Mutuswami (1997) and Jackson and van den Nouweland (2005). In particular, in the first, the formation and stability of social networks are modeled when agents choose to maintain or destroy links using the notion of pairwise stability. We follow Jackson and Wolinsky (1996) and Jackson and van den Nouweland (2004) to characterize the stable and the strongly stable networks.

Asides from these theoretical articles, there is also more and more literature that applies the theory of economic networks to models of oligopoly. In particular, the present article is closely related to Belleflamme and Bloch (2004). They have analyzed the collusive network of market-sharing agreements among firms, but they do not take into account the existence of antitrust authorities. Therefore, their results may be limited under those circumstances. They find that, in a stable network, there exists a lower bound in the size of collusive alliances. Moreover, when that threshold is equal to one, the set of isolated firms is composed, at most, by only one firm. These results are in contrast with the results of the current article. Under the presence of the antitrust authority, we are not able to define that lower bound and, ultimately, this fact implies that more competitive structure are possible to sustain in a such case.

Another application of network economics is networks and crime. Two recent articles related to the present one are Calvó-Armengol and Zenou (2004) and Ballester, Calvó-Armengol, and Zenou (2006).

Calvó-Armengol and Zenou (2004) study the impact of the network structure and its geometric details on individual and aggregate criminal behavior. Specifically, they provide a model of networks and crime, where the expected cost of committing criminal offenses is shaped by the network of criminal partners. Ballester, Calvó-Armengol, and Zenou (2006) further develop this approach. For their main results, they relate individual equilibrium outcomes to the players' positions in the network and also characterize an optimal network-based policy to disrupt the crime. In these articles, the network formation game is analyzed. This approach is different from ours. That is, we dispense with the specifics of the noncooperative game, and we model a notion of what is stable (a fixed-network approach). The other difference is the kind of externalities that one link entails. In those articles, the competition among criminals for the booty acts as a negative externality. Additionally, they assume that the criminal connections transmit to players (criminals) the necessary skill to undertake successful criminal activities, that is, a positive externality. Specifically, higher the criminal connections, lead to a lower individual probability of being caught. However, these assumptions are in contrast with assumptions in the present article about the externalities of signing a new agreement. Namely, we assume that more agreements increase the "booty" as long as the individual profits are a decreasing function in the number of active firms in the market (positive externality). On the other hand, each link entails a negative externality. As the number of agreements increases, the probability of being discovered also increases.

Regarding the collusion literature, after the seminal contribution of Stigler (1950), collusive cartels have been extensively studied. For an excellent reference of this literature, see Vives (2001).

As the present article, there are a number of articles that study the effect of antitrust policy on cartel behavior. Among others, we can mention Block et al. (1981) as the first systematic attempt to estimate the impact of antitrust enforcement on horizontal minimum price fixing. Their model explicitly considers the effect of antitrust enforcement on the decision of firms within an industry to fix prices collusively. They show that a cartel's optimal price is an intermediate price (between the competitive price and the cartel's price in absence of antitrust authority) and that this intermediate price depends on the levels of antitrust enforcement efforts and penalties.[3]

However, the interest for studying the effect of the antitrust policy on the collusive behavior has reemerged. Harrington (2004) and Harrington (2005) explore how detection affects cartel pricing when detection and penalties are endogenous. Firms want to raise prices but not suspicions that they are coordinating their behavior. In Harrington (2005), by assuming that the probability of detection is sensitive to price changes, he shows that the steady-state price is decreasing in the damage and in the probability of detection. These results are in line with results in the current article. However, he finds a long-run result of neutrality with respect to fixed penalties. Harrington (2004) studies the interaction of internal cartel stability and detection avoidance. One important result that he finds is the perverse effect of the antitrust law. The risk of detection and penalties can serve to stabilize a cartel and thereby allow it to set higher prices.

The main difference between the current article and the previous articles is that we study the impact of the antitrust policy on the structure of collusive agreements.

The outline of this article is as follows. Section 2 presents the model of market-sharing agreements and provides general definitions concerning networks. Section 3 characterizes the stable and strongly stable collusive networks in a symmetric context.

[3] Additionally, for example, Besanko and Spulber (1989 and 1990) with a different approach, use a game of incomplete information where the firms' common cost is private information and neither the antitrust authority nor the buyers observe the cartel formation. They find that the cartel's equilibrium price is decreasing in the fines. LaCasse, 1995 and Polo, 1997 follow this approach.

Section 4 studies the set of pairwise stable and strongly stable networks under different levels of antitrust enforcement. Furthermore, this section analyzes the impact of the antitrust authority over competition. The article concludes in Section 5. All proofs are relegated to the Appendix.

2 The Model

Firms

The model consists of N risk-neutral and symmetric firms indexed by $i = 1, 2, ..., N$. Each firm is associated to a market, that is, its home market. Markets are assumed symmetric. We are considering that each firm has incentives to enter into all foreign markets. Nevertheless, firm i does not enter into foreign market j, and vice versa, if a reciprocal market-sharing agreement exists between them. A reciprocal market-sharing agreement is an agreement whereby two firms agree not to enter each other's market.[4]

Let $g_{ij} \in \{0, 1\}$ denote the existence of an agreement between firms i and j. Thus, $g_{ij} = 1$ means that firm i has signed an agreement with firm j and vice versa.

Let n_i be the number of active firms in market i and m_i be the number of agreements formed by firm i. That is, $n_i = N - m_i$.

Let $\pi_j^i(\cdot)$ be the profits of firm i on market j. Firm i has two sources of profits. Firm i collects profits on its home market, $\pi_i^i(n_i)$, and on all foreign market where there does not exist an agreement, $\sum_{j, g_{ij}=0} \pi_j^i(n_j)$.

The symmetric firm and symmetric market assumptions allow us to write $\pi_j^i(\cdot) = \pi(\cdot)$. Therefore, the total profits of firm i can be written as follows:

$$\Pi^i = \pi(n_i) + \sum_{j, g_{ij}=0} \pi(n_j) \tag{1}$$

It is assumed that firms have limited liability, that is, $\Pi^i \geq 0$ is the maximum amount that the firm could pay in case a penalty were imposed by an antitrust authority.

Properties of profit functions

This article appeals to some properties for profit functions used by Belleflamme and Bloch (2004), henceforth BB. The profit functions satisfy the following properties:

Property 1: Individual profits are decreasing in the number of active firms in the market, $\pi(n_i - 1) - \pi(n_i) \geq 0$.

[4] It is assumed that these agreements are enforceable.

Property 2: Individual profits are log-convex in the number of active firms in the market, $\frac{\pi(n_i-1)}{\pi(n_i)} \geq \frac{\pi(n_i)}{\pi(n_i+1)}$.[5]

It is important to note that Property 1 is satisfied in the most standard oligopoly models. In spite of the fact that Property 2 is more restrictive than Property 1, BB provide sufficient conditions under which this property holds in a symmetric Cournot oligopoly context.[6] Moreover, they are satisfied by specific families of inverse demand functions, such as the iso-elastic and the exponential inverse demand functions.

The Antitrust Authority

We define an antitrust authority (AA) as a pair $\{\alpha, F(\Pi)\}$, where $\alpha \in [0,1)$ is the constant probability that a market-sharing suit is initiated, and $F(\Pi) \geq 0$ represents the monetary penalty that a firm must pay if it is convicted of market-sharing agreements. $F(\Pi)$ is a function that depends on the profits (Π) that a convicted firm ends up having. In fact, we assume that the AA sets the penalty equal to the total profits that a guilty firm ends up getting. That is, $F(\Pi) = \Pi$.[7]

The technology is such that when the AA inspects, if there is a market-sharing agreement, then the AA detects it. Moreover, the AA also identifies the two firms involved in the agreement. That is, if a firm is sued for making a market-sharing agreement, the AA is assumed to be able to detect, without error, whether a market-sharing agreement has occurred. Moreover, if it has occurred, the AA can detect the firms that signed that agreement. In such a case, both firms are penalized, and each must pay $F(\Pi) = \Pi^i$.

In the economic literature of optimal enforcement, fines are usually assumed as being socially costless. Therefore, when the AT seeks to deter collusion, the fines should be set at the maximum level in order to minimize the inspection cost.[8] An implication of this is that the fines need not to be related to the illegal profits or to the harm that the offenders caused. They only need to be as high as it is possible in order to deter collusion. This implication holds as long as there are not legal errors in the detection process (false convictions) or as long as the fines do not imply bankruptcy to convicted firms.

Regarding the inspection process, I assume that antitrust authorities have *constant* and *exogenous budgets* that allow them to inspect a fixed number of firms, that is, $\alpha \in [0,1)$ is a constant and exogenous probability of inspection. It can be also interpreted

[5]It can be proved that when profit are long-convex, they are also convex. That is, $\frac{\pi(n_i-1)}{\pi(n_i)} \geq \frac{\pi(n_i)}{\pi(n_i+1)} \Longrightarrow \pi(n_i-1) - \pi(n_i) \geq \pi(n_i) - \pi(n_i+1)$

[6]Let $P(Q)$ be the inverse demand function. In a symmetric Cournot oligopoly with homogeneous products, individual profits are decreasing in n if costs are increasing and convex and $E(Q) = \frac{QP''(Q)}{P'(Q)} > -1$. In this context, Property 3 is satisfied if costs are linear, $E(Q) > -1$ and $E'(Q) \geq 0$.

[7]See Roldán, 2008 for a detailed discussion.

[8]This holds when firms are risk-neutral.

as a surprise inspection policy, that although it may be effective,[9] it is not an usual practice.

The technology is such that when the AA inspects, if there exists a market-sharing agreement, then the AA detects it. Moreover, the AA also identifies the two firms involved in the agreement. That is, if a firm is sued for making a market-sharing agreement, the AA is assumed to be able to detect, without error, whether a market-sharing agreement has occurred. Moreover, if it has occurred, the AA can detect the firms that signed that agreement. In such a case, both firms are penalized, and each must pay $F(\Pi) = \Pi^i$.

The antitrust policy and the organization of collusive agreements

I will show how the organization of collusive conspiracy interacts with the enforcement policy. In particular, we will restrict our attention to the interaction between the structure of illegal agreements and the probability of being *detected*.

Given the technology of inspection assumed in this article, when a firm i forms a new market-sharing agreement, it will increase its probability of being detected. That is, the probability of firm i being caught by the AA depends not only on whether firm i is inspected but on whether any other firm with which firm i has a link, is also inspected.[10] Therefore, firm i will not be detected if i is not inspected *and* if j, that has an agreement with i, is not inspected. That is, $\Pr(\text{Detected } i) = 1 - \Pr(\text{No Detected } i)$, where

$$\Pr(\text{No Detected } i) = \Pr\left(\text{No inspected } i \bigcap_{\substack{j\neq i \\ g_{ij}=1}} \text{No inspected } j\right)$$

Or, equivalently,[11] $\Pr(\text{No Detected } i) = (1-\alpha) \prod_{\substack{j\neq i \\ g_{ij}=1}} (1-\alpha)$. That is:

$$\Pr(\text{No Detected } i) = (1-\alpha)^{N-n_i+1} \tag{2}$$

Therefore, the probability of being detected depends on how many agreements firm i has signed, that is, $m_i = N - n_i$.[12] Note that, as the number of agreements $m_i = N - n_i$ increases, $\Pr(\text{Detected } i)$ goes to one. On the other hand, as $m_i = N - n_i$ goes to zero, $\Pr(\text{Detected } i) \to \alpha$.[13]

[9] Friederiszick and Maier-Rigaud (2007) argue that "surprise inspections are by far the most effective and sometimes the only means of obtaining the necessary evidence...."

[10] We only consider the immediate link.

[11] It is assumed that events "No inspected i" and "No inspected j" are independent each other.

[12] Let us observe that the number of terms in the operator $\prod$ is $m_i = N - n_i$.

[13] Observe that the $\Pr(\text{ Detected } i) = 1 - \Pr(\text{No Detected } i)$ is increasing and concave in the number of agreements signed. That is, as $m_i = N - n_i$ increases, the probability of being detected

From the AA's point of view, the structure of relationships described by $m_i = N - n_i$ generates scale economies on detection as

$$\Pr(\text{ Detected } i) = 1 - (1-\alpha)^{N-n_i+1} > \Pr(\text{ Inspected } i) = \alpha$$

Incentives to form an agreement

An essential part of the model is the firm's incentive to form an agreement. Assume that firm i has formed $m_i = N - n_i$ agreements, but has not yet formed an agreement with a firm j, that is, $g_{ij} = 0$. Then, by using expressions (1) and (2), we compute firm i's expected profits as:

$$(1-\alpha)^{N-n_i+1}\,\Pi^i + \left(1 - (1-\alpha)^{N-n_i+1}\right)\left[\Pi^i - F(\Pi)\right] \tag{3}$$

where $F(\Pi) = \Pi^i$ and $\Pi^i = \pi(n_i) + \pi(n_j) + \sum_{k \neq j, g_{ki}=0} \pi(n_k)$.

Now, if firm i decides to form a link with firm j, its expected profits will be

$$(1-\alpha)^{N-n_i+2}\,\Pi^i + \left(1 - (1-\alpha)^{N-n_i+2}\right)\left[\Pi^i - F(\Pi)\right] \tag{4}$$

but now, $\Pi^i = \pi(n_i - 1) + \sum_{k \neq j, g_{ki}=0} \pi(n_k)$.

By subtracting (3) from (4), we obtain firm i's incentive to form an agreement with firm j as:

$$\Delta\Pi^i_j = (1-\alpha)^{N-n_i+1}\left[\pi(n_i - 1) - \pi(n_i) - \pi(n_j) - \alpha\left(\pi(n_i - 1) + \sum_{k \neq j, g_{ki}=0} \pi(n_k)\right)\right] \tag{5}$$

Let $J^i_j(n_i, n_j, n_k; \alpha)$ denote the bracket expression in (5). It can then be rewritten as

$$\Delta\Pi^i_j = (1-\alpha)^{N-n_i+1}\, J^i_j(n_i, n_j, n_k; \alpha) \text{ such that } g_{ki} = 0$$

It is worth noting that when $\alpha = 0$ firm i's incentive to form a market-sharing agreement with firm j only depends on the characteristics of markets i and j. When

increases. However, it increases at a decreasing rate.

an antitrust authority exists, however, $\Delta\Pi_j^i$ will also depend on the characteristics of all markets k in which firm i is active.[14]

We are interested in the sign of $\Delta\Pi_j^i$ because it is what is relevant for deciding whether or not one more link is formed. That is, if $\Delta\Pi_j^i \geq 0$, firm i has an incentive to form an agreement with j.

Therefore, when $\alpha \neq 1$, $\Delta\Pi_j^i \geq 0$ only if $J_j^i(n_i, n_j, n_k; \alpha) \geq 0$. Hence, in the following, we will focus only on $J_j^i(n_i, n_j, n_k; \alpha)$.

Forming one more link has several conflicting consequences. From firm i's point of view, note that when a link is formed between firms i and j, firm j agrees not to enter market i. Therefore, the number of active firms in market i will decrease, and it increases its profits by $\pi(n_i - 1) - \pi(n_i)$. Given the reciprocal nature of this agreement, firm i does not enter market j, either. Then, firm i loses access to foreign market j, and decreases its profits by $\pi(n_j)$. Additionally, if firm j is inspected, and it is inspected with probability α, firm i will lose $\pi(n_i - 1) + \sum\limits_{k \neq j, g_{ki}=0} \pi(n_k)$.

Note that, as $\pi(\cdot)$ is a decreasing function, when n_j decreases, it decreases the incentive to lose a more profitable market by forming a link. Then, J_j^i is *increasing* in n_j.

Likewise, J_j^i is *increasing* in n_k. As n_k gets smaller, the expected costs of signing an agreement with j become greater.[15] It thus decreases the incentive to form a collusive agreement.

On the other hand, the relationship between J_j^i and n_i is ambiguous. As $\pi(\cdot)$ is a convex function,[16] when the number of competitors in its home market decreases, $\pi(n_i - 1) - \pi(n_i)$ increases, and thus J_j^i increases. However, in such a case, $\alpha\pi(n_i - 1)$ increases, that is, the expected cost of forming an agreement increases. As a result, this reduces the incentive to form it.

Concerning the antitrust policy, when the probability of inspection α increases, J_j^i *decreases*, because it increases the expected cost of forming a link.

To sum up, the relationship between firms and the competition authority is as follows: given the antitrust policy $\{\alpha, F(\Pi)\}$, firms compute the incentives to form agreements and then decide whether or not to form an agreement. Firms form them if they yield positive profits after expected penalties from signing market-sharing agreements. If an inquiry is opened, and if a firm is convicted of forming a market-sharing agreement, it must pay $F(\Pi) = \Pi^i$.

[14] We just consider the case when $m_i = N - n_i \neq 0$. However, when firm i is isolated, that is, $m_i = N - n_i = 0$, the firm i's incentive to form an agreement is slightly different from (5). That is, $\Delta\Pi = \pi(N-1)(1-\alpha)^2 - \pi(N) - \pi(n_j) - \sum\limits_{k \neq j, g_{ki}=0} \pi(n_k)\left(1 - (1-\alpha)^2\right)$.

[15] The expected cost is $-\left(\alpha \sum\limits_{k \neq j, g_{ki}=0} \pi(n_k)\right)$.

[16] See Footnote 5.

Background definitions

In this part, we provide some definitions that will be useful in describing and analyzing the model.

That is, we are considering firms that enter into *bilateral* relationships with each other, that is, market-sharing agreements are *bilateral* agreements and the set of them gives rise to a collusive network g.

We thus introduce some notations and terminology from graph theory to study the collusive network.

Networks Let $N = \{1, 2, ..., N\}$, $N \geq 3$ denote a finite set of identical firms.

For any $i, j \in N$, the pairwise relationship or link between the two firms is captured by a binary variable $g_{ij} \in \{0, 1\}$, defined as before.

A network $g = \left\{ (g_{ij})_{i,j \in N} \right\}$ is a description of the pairwise relationship between firms.

Let $g + g_{ij}$ denote the network obtained by adding link ij to an existing network g and denote by $g - g_{ij}$ the network obtained by deleting link ij from an existing network g.

Some networks that play a prominent role in our analysis are the following two: the complete network and the empty network.

The *complete network*, g^c, is a network in which $g_{ij} = 1, \forall i, j \in N$. In contrast, the *empty network*, g^e, is a network in which $g_{ij} = 0, \forall i, j \in N, i \neq j$.

Formally, a firm i is isolated if $g_{ij} = 0, \forall j \neq i$ and $\forall\ j \in N$.

Paths and Components A *path* in a network g between firms i and j is a sequence of firms $i_1, i_2, ..., i_n$ such that $g_{ii_1} = g_{i_1 i_2} = g_{i_2 i_3} = ... = g_{i_n j} = 1$. We will say that a network is *connected* if there exists a path between any pair $i, j \in N$.

A *component* g' of a network g is a maximally connected subset of g. Note that from this definition, an isolated firm is not considered a component.

Let $m_i(g')$ denote the number of links that firm i has in g'.

A component $g' \subset g$ is *complete* if $g_{ij} = 1$ for all $i, j \in g'$. For a complete component g', $m_i(g') + 1$ denote its size, that is, it is the number of firms belonging to g'.

The next figure represents a network with two complete components and an isolated firm.

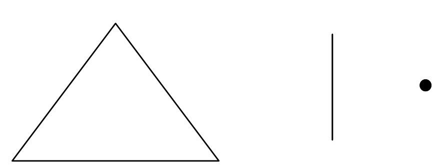

Figure 2: Two complete components of size 3 and 2 and one isolated firm.

Stable collusive networks Our interest is to study which networks are likely to arise. As a result, we need to define a notion of stability. In the present article, we always use a notion of *pairwise* stability.

Pairwise stable networks The following approach is taken by Jackson and Wolinsky (1996). A network g is *pairwise stable* if and only if: (i) $\forall i, j \in N$ such that $g_{ij} = 1$, $\Pi^i(g) \geq \Pi^i(g - g_{ij})$ and $\Pi^j(g) \geq \Pi^j(g - g_{ij})$; and (ii) $\forall i, j \in N$ such that $g_{ij} = 0$, if $\Pi^i(g + g_{ij}) > \Pi^i(g)$ then $\Pi^j(g + g_{ij}) < \Pi^j(g)$.

In terms of our model, a network g is said to be pairwise stable if and only if:

$$\text{(i)} \quad \forall i, j \ s.t. \ g_{ij} = 1, \ \begin{cases} J_j^i(n_i + 1, n_j + 1, n_k; \alpha) \geq 0 \\ J_i^j(n_j + 1, n_i + 1, n_k; \alpha) \geq 0 \end{cases}$$

$$\text{(ii)} \quad \forall i, j \ s.t. \ g_{ij} = 0, \ \begin{cases} \text{if } J_j^i(n_i, n_j, n_k; \alpha) > 0 \\ \text{then } J_i^j(n_j, n_i, n_k; \alpha) < 0 \end{cases}$$

It is worth noting that the first part of the definition requires that no firm would want to delete a link that it serves. In other words, any firm has the discretion to *unilaterally delete* the link. This contrasts with the second part of the definition which means that the consent of both is necessary to form a link. That is, *forming a* link *is* a *bilateral decision.*

The above stability notion is a relatively weak criterion in the sense that it provides broad predictions and the firm's deviations are constrained. A pairwise stability criterion only considers deviations from a single link at a time.[17] Furthermore, the pairwise stability notion considers only deviations by a pair of players at a time.[18]

Nevertheless, that criterion provides a test to eliminate the unstable networks and it should be seen as a necessary but not sufficient condition for a network to be stable.

Strongly pairwise stable networks In order to obtain a stronger concept of stability, we allow deviations by coalitions of firms. We allow firms to delete some or all market-sharing agreements that they have already formed.

We say that a network is pairwise strongly stable if it is immune to deviations by coalitions of two firms. As BB do, we consider the simultaneous linking game introduced by Myerson (1991). Each firm i chooses the set s_i of firms with which it wants to form a link. As a result, $g_{ij} = 1$ if and only if $j \in s_i$ and $i \in s_j$. Let $g(s_1, s_2, ..., s_n)$ denote the network formed when every i chooses $s_{i.}$.

A strategy profile $\{s_1^*, s_2^*, ..., s_n^*\}$ is a pairwise strong Nash equilibrium of the game if and only if it is a Nash equilibrium of the game and there does not exist a pair of firms i and j and strategies s_i and s_j such that $\Pi^i\left(g\left(s_i, s_j, s_{-ij}^*\right)\right) \geq \Pi^i\left(g\left(s_i^*, s_j^*, s_{-ij}^*\right)\right)$ and $\Pi^j\left(g\left(s_i, s_j, ..., s_{-ij}^*\right)\right) \geq \Pi^j\left(g\left(s_i^*, s_j^*, ..., s_{-ij}^*\right)\right)$ with a strict inequality for one of the

[17] On the contrary, for example, it is possible that a firm would not benefit from forming a single link but would benefit from forming several links simultaneously.

[18] It could be that larger groups of player can coordinate their actions in order to all be better off.

two firms. A network g is strongly pairwise stable if and only if there exists a pairwise strong Nash equilibrium of the game $\{s_1^*, s_2^*, ..., s_n^*\}$ such that $g = g(s_1^*, s_2^*, ..., s_n^*)$.

It is possible to prove that any strong pairwise stable network is pairwise stable. The strong stability notion can thus be thought of as sufficient condition for stability.

3 Stable collusive networks characterization

In this section, we will characterize pairwise stable and strongly pairwise stable networks under the presence of the AA in a symmetric context. Let us recall that the notion of pairwise stability might be thought of as a necessary but not sufficient condition for stability and that the strong pairwise stable criterion provides a sufficient requirement for a network to be stable over time. Also, recall that any strong pairwise stable network is pairwise stable.

Pairwise stable collusive network

Two results from BB's model still hold in the current context. The following two provide the necessary conditions on pairwise stability.

Result 1 *Under Property 1, if network g is pairwise stable, then $\forall i, j \in N$ such that $g_{ij} = 1$, $n_i(g) = n_j(g)$.*

In the following, we simply use $n_i(g) = n_i$. That is, two firms are connected by a market-sharing agreement in a stable network if they have the same number of competitors in their home markets. If not, the link is not served. That is, if $n_i \neq n_j$, the firm in the less profitable market (with a larger number of competitors in its home market) does not have an incentive to lose access to a more profitable one (with a smaller number of competitors) by signing a market-sharing agreement.

Result 2 *Under Property 1 and 2, if network g is stable, then any component g' of g is complete. Moreover, if there is more than one component, they have different sizes.*

Then, by Result 2, a pairwise stable network can be decomposed into complete alliances of different sizes. Then, if a set of firms is linked, all of them must be linked by a market-sharing agreement among themselves, that is, complete components. Moreover, if a pairwise stable network has more than one component, they have different sizes. As BB already establish, the intuition underlying this result is due to free riding. If a firm i has signed more agreements than other firms, market i is a very profitable one. Therefore, the other firms will not want to form a link with firm i because they do not want to lose access to a profitable market. In other words, the other firms *ride free* on the agreements signed by firm i. By extending this argument, we say that firms in smaller alliances (with a larger number of competitors in their home markets) *free ride* on the agreements signed by firms in larger ones (with smaller number of competitors), as any firm belonging to a small alliance has no incentive to form an agreement with a firm that belongs to a large one.

Now, let us observe that Results 1 and 2 are stated by using Property 1 and Property 2. Hence, in the absence or under the presence of the AA, both results are always true. Additionally, let us note that Results 1 and 2 talk about *linked* firms and how they are linked. Then, in this context, two questions become relevant. Does the AA have any impact on the *size* of the linked firms' alliances? Does the AA have any impact on the set of firms that remain without links? The answer to that is interconnected. In our setting, the competition authority affects the net expected profits from entering into a market-sharing agreement and in turn impacts on the decision whether or not a link is formed. Therefore, from the definition of J_j^i, $g_{ij} = 1$ only if

$$(1-\alpha)\,\pi\,(n-1) \geq 2\pi\,(n) + \alpha \sum_{k \neq h, ghk=0} \pi\,(n_k)\,, \quad \forall\ h = i, j \tag{6}$$

Let us see the impact of the AA on the decision to participate in a collusive agreement.

In the absence of the AA, that is, $\alpha = 0$,[19] the above inequality becomes $\pi\,(n-1) > 2\pi\,(n)$. Therefore, by log-convexity, it is possible to guarantee the existence of a number $n^* = N - m^*$ such that $\pi\,(n^*-1) \geq 2\pi\,(n^*)$. $m^* = N - n^*$ is thus interpreted as the minimal number of agreements that a firm already has to have in order to form an additional one. In the absence of a competition authority, there exists a lower bound on the size of collusive alliances, which does not depend on g. Moreover, when $m^* = 1$, the number of isolated firms is at most 1.

In contrast, under the presence of the AA, that is, $\alpha \neq 0$, we are not able to reach a unique lower bound. From (6) we can see that the maximal number of competitors that assures that the condition holds depends on α and on g. Moreover, we will see that if in a stable network the alliance of minimum size is equal to two, that is, $m^* = 1$, it does not impose any restriction on the set of isolated firm. The consequences of that over competition will be discussed further in Section 4.

Consider the following network g that can be decomposed into distinct complete components, g_1,...,g_L, of different sizes, that is, $m\,(g_l) \neq m\,(g_{l'})$, $\forall l, l'$.

Let us define $m\,(g_h^*) := \min\{m\,(g_1)\,, ..., m\,(g_L)\}$. That is, g_h^* is the smallest component of a network g, whose size is $m\,(g_h^*) + 1$.

The next lemma shows a sufficient condition on pairwise stability in our collusive context.

Lemma 1 *Given a network g that can be decomposed into a set of isolated firms and different complete components, g_1,...,g_L, of different sizes $m\,(g_l) \neq m\,(g_{l'})$, $\forall l, l'$, if firm $i \in g_h^*$ does not have incentives to cut a link with a firm inside its alliance, then any $j \in g_l$ will not have incentives to cut a link with a player inside its component for all $g_l \neq g_h^*$.*

[19] It is the Belleflamme and Bloch's setting.

The lemma provides a condition to check whether a firm has incentives to renege on one agreement. Then, given a network g, it is sufficient to verify what happens inside the smallest component. The intuition is as follows. A firm that belongs to the smallest component has two disadvantages: (i) it has a larger number of competitors in its home market than any firm that belongs to a greater alliance, and (ii) if the antitrust authority detects its agreements, it loses profits on markets where it does not collude,[20] and they are larger than the same kind of profits of a firm that belongs to a larger cartel.[21] Therefore, if any firm $i \in g_h^*$ has no incentive to renege on one agreement, no other linked firm will have it.

By combining the previous Results and the Lemma, we state the following.

Proposition 1 *A network g is pairwise stable if and only if it can be decomposed into a set of isolated firms and distinct complete components, g_1,...,g_L of different sizes $m(g_l) \neq m(g_{l'})$, $\forall l, l'$ such that no isolated firm has an incentive to form a link with another isolated one and no firm i that belongs to the smallest component has an incentive to cut a link with a firm inside it.*

The above Proposition provides the characterization of the pairwise stable networks in the symmetric context when the AA exists. Note that the Proposition holds for all $m(g_h^*) \geq 1$.

Additionally, in Section 4, we will see that in this context, the pairwise stable network always exists.

It is important to note that the AA imposes a change in the minimal size of the components, and that it does not restrict the set of isolated firms. In the absence of the AA, that is, the BB's setting, a network is stable if its alliances are large enough. That is, the complete components have to reach a minimal size, that is, m^*.

However, under the presence of the AA, that threshold, that is, $m(g_h^*)$, depends on α and on g.

By rewriting (6), we obtain the following:

$$\frac{\pi(n-1)}{\pi(n)} \geq \frac{2}{(1-\alpha)} + \frac{\alpha \sum\limits_{k \neq h: g_{hk}=0} \pi(n_k)}{(1-\alpha)\pi(n)} \geq 2, \quad \forall\ h = i, j \text{ such that } g_{ij} = 1 \tag{7}$$

In spite of the fact that $m(g_h^*)$ depends on particular conditions, it is easy to see that $m(g_h^*) \geq m^*$. Nevertheless, in Section 4, we will show that this is not necessarily a perverse effect of the AA because $m(g_h^*) \geq 1$ does not put any restriction on the set of isolated firm.

[20] That is, $\sum\limits_{k, g_{ki}=0} \pi(n_k)$.

[21] Assume, for simplicity, that there are only two complete alliances and that $i \in g_l$ and $j \in g_{l'}$, where $m(g_l) < m(g_{l'})$. Then, $\sum\limits_{k, g_{ki}=0} \pi(n_k) = [m(g_{l'}) + 1]\pi(n_j) > \sum\limits_{k, g_{kj}=0} \pi(n_k) = [m(g_l) + 1]\pi(n_i)$.

Pairwise strongly stable collusive network

We refine the set of stable networks by using the strong stability condition. Now we allow firms to delete a subset of links already formed and we will study when a firm has no incentive to renege on its agreements. This point is very important in our context because a network composed by large alliances will be difficult to sustain.

Proposition 2 *A network g is pairwise strongly stable if and only if it is pairwise stable and no firm prefers to cut all its agreements at once, that is,*

$$(1-\alpha)^{N-n+1}\pi(n) \geq \pi(N) + (N-n)\pi(n+1) + \sum_{k,gi=0} \pi(n_k)\left(1-(1-\alpha)^{N-n+1}\right),$$

$$\forall\ n = N-m+1 \text{ and } \forall\ m = m(g_l) \tag{8}$$

Accordingly, the fact that a firm has no incentives to renege on all of its links at once is a sufficient condition for strong stability. To see this, assume that a firm reneges on one of its agreements. Then, it gains access to a market whose profits are at least equal to the profit it makes on its home market after cutting a link. Therefore, if a firm has an incentive to cut one agreement, the most profitable deviation for it is to renege on all its agreements at once.

Therefore, in a strongly stable network, component sizes satisfy a more demanding condition.

It is worth remarking that a strongly stable network may fail to exist. Nonetheless, one important advantage of the strong criterion is to provide a more accurate prediction of which network structures will prevail.

Examples

Example 1 *Pairwise Stable Network for $\alpha = 0$ and $\alpha \neq 0$. Cournot competition with exponential inverse demand function $P(Q) = e^{-Q}$*

When the inverse demand function is $P(Q) = e^{-Q}$, we can compute the equilibrium profits as $\pi(n) = e^{-n}$.

In the absence of the AA, that is, $\alpha = 0$, the pairwise stability condition (7) becomes

$$\frac{\pi(n-1)}{\pi(n)} = e \geq 2, \forall n$$

Therefore, any two firms have incentives to form a link. Therefore, $m^* = 1$ and any network with complete components of different sizes with at most one isolated firm is pairwise stable.

In contrast, when AA exists, that is, $\alpha \neq 0$ that is no longer true. Assume, for example, $N = 7$ and $\alpha = 0.025$. In such a context, the following is one network configuration that belongs to the set of the pairwise stable networks:

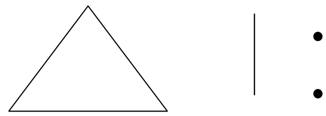

Figure 3: Stable network, $N = 7$ and $\alpha = 0.025$.

Let us observe that in this case, $m(g_h^*) = 1$, and the number of isolated firms in that stable network is greater than one. This result is in a sharp contrast to the prediction established in the absence of the AA.

We can easily check the sufficient conditions for pairwise stability: (i) no firm in the smallest component wants to cut a link that it serves because it is profitable to maintain it. That is, (7) holds.[22] Also, (ii) for any isolated firm, it is true that $\frac{\pi(6)}{\pi(7)} < \frac{2}{(1-\alpha)} + \frac{\alpha(3\pi(5)+2\pi(6))}{(1-\alpha)\pi(7)}$.

Example 2 *Pairwise Stable Network and Strongly Stable Network for* $\alpha = 0$ *and* $\alpha \neq 0$. *Cournot competition with exponential inverse demand function* $P(Q) = e^{-Q}$

As stated above, in this competitive context, $\pi(n) = e^{-n}$.

Now, assume $N = 5$. The following table depicts the set of pairwise stable (ps) and pairwise strongly stable (pss) networks for $\alpha = 0$ and for $\alpha = 0.03$.

First of all, it is useful to clarify some notations there. In the table, the complete network is represented by $\{5\}$, and, for example, $\{3,1,1\}$ denotes a network decomposed into two isolated firms and one complete component of size three.

Table 1

α	**Set of ps networks**	**Set of pss networks**
$\alpha = 0$	$\{3,2\},\{4,1\},\{5\}$	$\{3,2\}$
$\alpha = 0.03$	$\{3,1,1\},\{4,1\},\{5\}$	It fails to exist

When $\alpha = 0$, any two firms have an incentive to form a market-sharing agreement, as $\frac{\pi(n-1)}{\pi(n)} = e \geq 2, \forall n$. In other words, for $\alpha = 0$, $m^* = 1$.

By applying the strong-stability condition, we obtain that, when $\alpha = 0$, only the network whose components have size 3 and 2 is strongly stable.

Let us note, from Table 1, that the strong criterion selects a subset of stable networks, which allows us to improve our prediction about which networks prevail over time.

Now, let us observe that, for $\alpha = 0.03$, $m(g_l^*) = 2 > m^* = 1$. In spite of this fact, it is easy to see that the network $\{3,1,1\}$ entails more competition than $\{3,2\}$.

Additionally, this example illustrates that, in some circumstances, the strongly stable network fails to exist and that every network is defeated by some other network, which only leads to a cycling of these events.

[22] See Roldán, 2008 for all calculation details.

4 The Antitrust Authority and the set of stable collusive networks

In our setting, the presence of the antitrust authority imposes a cost to each formed link, and as a result, the expected gain of being a part of a collusive agreement may not be positive.[23] That is, the expected sanction imposed by the AA affects the incentive participation constraint of each potential alliance's member and in turn changes the set of possible network structures that can arise.

Given the network characterization of the previous section, we now analyze which kinds of stable networks can be sustained at different levels of the antitrust enforcement.

The set of pairwise stable networks

First of all, a complete network is always pairwise stable for sufficiently low values of α. Let us define $\alpha^c := 1 - \frac{2\pi(2)}{\pi(1)}$.

Proposition 3 *The complete network g^c is pairwise stable if and only if $\alpha \leq \alpha^c$.*

Being a part of a collusive agreement entails positive benefits. To serve a link increases the profits of firms that participate in it, that is, $\pi(n)$ is decreasing in n. Therefore, the complete network will be pairwise stable as long as its costs, that is, the expected sanction are sufficiently low.

Second, the empty network arises as pairwise stable for sufficiently high values of α. Let us define $\alpha^e(N) := 1 - \left[\frac{N\pi(N)}{[\pi(N-1)+(N-2)\pi(N)]}\right]^{\frac{1}{2}}$, for $\forall N \in [3, \infty)$ and $\alpha^e(N) < 1$.

Proposition 4 *For $\forall N \in [3, \infty)$, the empty network g^e is pairwise stable if and only if $\alpha > \alpha^e(N)$.*

For an isolated firm, $\alpha^e(N)$ is the threshold from which it has no incentive to participate in an agreement when all other firms also remain isolated. When $\alpha > \alpha^e(N)$, the expected costs of forming a link are so high, relative to its benefits, that no two firms will sign an agreement.

Moreover, observe that $\alpha^e(N)$ is strictly decreasing in N. That is, as N increases, the "loot" becomes less "attractive" (that is, $\pi(N)$ is decreasing in N), and the threshold will decrease as a result.

By straightforward computations, we can see that $\alpha^e(N) < \alpha^c$. Consequently, from the above Propositions, we claim the following:

---

[23] It is, $J_j^i := \pi(n_i - 1) - \pi(n_i) - \pi(n_j) - \alpha \left(\pi(n_i - 1) + \sum_{k \neq j, g_{ki}=0} \pi(n_k)\right), \forall i, j.$

Claim 1 *For $\alpha \in (\alpha^e(N), \alpha^c]$, g^e and g^c belong to the set of pairwise stable networks.*

From Proposition 3 and 4, we can state that pairwise stable networks always exist. That is, first, for $\alpha \leq \alpha^c$, the complete network belongs to the set of stable networks. Second, for $\alpha > \alpha^e(N)$, the empty network will be stable. And given that $\alpha^e(N) < \alpha^c$, for $\alpha \in (\alpha^e(N), \alpha^c]$, g^e and g^c arise as pairwise stable configurations.

When $\alpha \neq 0$, there exists a positive probability of being caught in a market-sharing agreement. Consequently, there exists a positive probability of losing profits not only in the market where the agreement is signed but also in markets in which the firm is active, that is in markets where the firm does not collude.

For firms in smaller alliances, the cost of forming a link becomes more significant relative to the benefits of doing so. That is, a firm i inside a small alliance does not have much to gain and has a lot to lose when one more link is made. More precisely, by signing an agreement, it gains $(1-\alpha)\pi(n_i - 1) - \pi(n_i)$, which decreases as the alliance becomes smaller;[24] and it not only loses the access to profits in the foreign market j, $\pi(n_j)$ but also loses, in expected terms, $\alpha \sum\limits_{k:g_{ik}=0} \pi(n_k)$.

Therefore, firms in smaller components are more sensitive to the antitrust enforcement.

The intuition provided above is summarized in the next Proposition. Before introducing it, let us define

$$\alpha^*(n_i) := \frac{\pi(n_i - 1) - 2\pi(n_i)}{\pi(n_i - 1) + \sum\limits_{k \neq j, gik=0} \pi(n_k)}$$

That is, at $\alpha^*(n_i)$ a firm i, with n_i competitors in its home market, is indifferent to forming a link, that is, $J_j^i = 0$. Therefore, when $\alpha > \alpha^*(n_i)$, then $J_j^i < 0$, and firms i and j do not sign a collusive agreement.

Proposition 5 *For firm $i \in g_1$ and firm $j \in g_2$ such that $m(g_1) < m(g_2)$, then $\alpha^*(n_1) < \alpha^*(n_2)$.*

From the Proposition it follows that the threshold is smaller for firms in smaller alliances (with larger number of competitors in their home markets). Then, as α becomes greater, the AA first tears down small alliances, that is, the smaller components are more sensitive to the antitrust policy. In the limit, firms must decide to form a very large alliance (complete network) or no alliance at all (empty network).

Proposition 6 *For $\alpha = \alpha^c > 0$, the only pairwise stable networks are g^e and g^c.*

Then, by setting $\alpha > \alpha^c$, the AA completely deters the formation of collusive agreements.

[24] Remember that the number of active firms is greater in smaller components.

The set of pairwise strongly stable networks

Now, we turn our attention to the notion of strongly stable networks and we answer which kinds of networks arise as the AA changes its enforcement level. From the previous section, we know that there will be some pairwise stable networks that will not be stable against changes in the agreements made by firms. By applying (8), we assert the following:

Proposition 7 *As α increases, firms in large components have more incentives to delete all links at once.*

That is, as α increases, the condition of strong stability is harder to sustain in larger components. In other words, faced with increasing α, a firm has to consider whether to maintain or to destroy its agreements. Therefore, the firm balances the pros and the cons of any decision. Namely, if a firm maintains its agreements, its benefits are $(1-\alpha)^{N-n+1}\left[\pi(n)+\sum_{k,gi=0}\pi(n_k)\right]$.

Let us note that these benefits decrease as the probability of inspection (α) increases, and the fall in the expected benefits is higher as $m = N - n$ increases.

Instead, if the firm decides to destroy all its agreements, it is not only not penalized now by the AA, but it will also gain access to markets where it was colluding before. In such a situation, it will make profits on all these new foreign markets; that is, $(N-n)\pi(n+1)$. Let us observe that these markets are more profitable as the number of competitors on them is smaller, that is, as $m = N - n$ is larger.

Therefore, firms belonging to larger alliances have more incentives to cut all their agreements at once as, the AA increases the cost of forming links.

Now let us consider the empty network under the strongly stable notion.

It is worth noting that if g^e is pairwise stable, it is also strongly pairwise stable, as the condition (8) is always satisfied for firms that remaining alone. That is, in an empty network, firms do not have any link, so the condition of not having incentives to renege on all agreements at once is redundant for any $i \in g^e$. Therefore, we claim that

Claim 2 $\forall\alpha > \alpha_e(N)$ *the empty network is always strongly pairwise stable.*

Accordingly, if for some $\alpha > \alpha_e(N)$ all alliances have been torn down by the antitrust policy, the only network configuration that exists is the empty one.

Examples

The following examples illustrate the changes that the AA imposes in the set of pairwise stable networks.[25]

[25] See Roldán, 2008 for all calculation details.

Example 3 *Pairwise stable (ps) networks. Cournot competition with exponential inverse demand function* $P(Q) = e^{-Q}$

Let us recall that in this context $\pi(n) = e^{-n}$. Assume that $N = 5$. The following table depicts the set of pairwise stable networks for different values of the antitrust policy.

Table 2

α	**Set of ps networks**
$\alpha \in [0; 0.015)$	$\{3,2\}, \{4,1\}, \{5\}$
$\alpha \in [0.015; 0.04)$	$\{3,1,1\}, \{4,1\}, \{5\}$
$\alpha \in [0.04; 0.065)$	$\{2,1,1,1\}, \{3,1,1\}, \{4,1\}, \{5\}$
$\alpha \in [0.065; 0.21)$	$\{1,1,1,1,1\}, \{3,1,1\}, \{4,1\}, \{5\}$
$\alpha \in [0.21; 0.25)$	$\{1,1,1,1,1\}, \{4,1\}, \{5\}$
$\alpha \in [0.25; 0.26)$	$\{1,1,1,1,1\}, \{5\}$
$\alpha > 0.26$	$\{1,1,1,1,1\}$

Thus, when α is sufficiently low (that is, $\alpha < 0.015$) the presence of the AA does not change the set of pairwise stable networks. However, when the antitrust enforcement is sufficiently high (that is, $\alpha > 0.26$) the only pairwise stable network is the empty one, as a result all, firms are active in all markets.

Consider now values for values of α between these two extreme cases. Although different configurations arise, the main features to be highlighted are the following two. First, when α increases, more structures that are competitive can be sustained through bilateral agreements. In particular, when α becomes greater, the smaller components are more sensitive to the antitrust policy. For example, when $\alpha \in [0.015; 0.04)$, the network structure $\{3,2\}$ is not stable because firms in smaller components have incentives to cut their agreements and the network $\{3,1,1\}$ becomes stable.[26] Second, as α increases the set of stable network configurations becomes more polarized. That is, in our analytical example, when $\alpha \in (0.25; 0.26)$, the empty or complete networks are the only possible stable network configurations. This can be understood because the AA imposes the costs of forming links, and reduces the profitability of each one. As a result, firms decide either to form more and more links, that is reduce the number of competitors in their home markets, in order to balance their benefits with their cost; or to form no link at all and thus avoid the costs levied by the AA.

The next example illustrates two special features of the strong criterion and the impact of the AA on the set of strongly stable networks.

Example 4 *Pairwise strongly stable (pss) networks. Cournot competition for exponential inverse demand function:* $P(Q) = e^{-Q}$

[26] Likewise, it is noteworthy that the graph $\{3,1,1\}$ is not pairwise stable in the BB's setting, that is, when $\alpha = 0$.

As in the last example, assume that $N = 5$. Given that a pairwise strongly stable network is always pairwise stable, it suffices to check the condition (8) for all network structures in Table 2 at different levels of the antitrust policy.

Table 3

α	**Set of ps networks**	**Set of pss networks**
$\alpha \in [0; 0.015)$	$\{3,2\},\{4,1\},\{5\}$	$\{3,2\}$
$\alpha \in [0.015; 0.04)$	$\{3,1,1\},\{4,1\},\{5\}$	it fails to exist
$\alpha \in [0.04; 0.065)$	$\{2,1,1,1\},\{3,1,1\},\{4,1\},\{5\}$	$\{2,1,1,1\}$
$\alpha \in [0.065; 0.21)$	$\{1,1,1,1,1\},\{3,1,1\},\{4,1\},\{5\}$	$\{1,1,1,1,1\}$
$\alpha \in [0.21; 0.25)$	$\{1,1,1,1,1\},\{4,1\},\{5\}$	$\{1,1,1,1,1\}$
$\alpha \in [0.25; 0.26)$	$\{1,1,1,1,1\},\{5\}$	$\{1,1,1,1,1\}$
$\alpha > 0.26$	$\{1,1,1,1,1\}$	$\{1,1,1,1,1\}$

First of all, the example clarifies that the possible set of stable networks is reduced by using the criterion of strong stability. However, the strongly stable network might fail to exist, and this is what happens for $\alpha \in [0.015; 0.04)$.

Second, the incentive to free ride and delete all links is higher in larger alliances. That is, when a firm that belongs to a large alliance cuts all its agreements at once, it will recover its access to more profitable markets than a firm belonging to a small component. In the example, the complete network $\{5\}$ and the stable network $\{4,1\}$ do not pass the strongly stable condition. By extending this argument, the empty network is the only strongly stable network for $\alpha > 0.065$.

Therefore, the antitrust policy is on the side of the competition as long as it gives firms in large alliances more incentives to renege on their agreements at once.

The AA and its effects on competition

From the previous analysis, we conclude that as α increases, the smaller alliances are first in being destroyed by the antitrust policy. In turn, the set of isolated firms expands.

Moreover, as α becomes larger, $m\left(g_h^*\right)$ also increases. From Proposition 7, however, we know that large alliances are harder to sustain.

Therefore, as α increases, the empty network, g^e, tends to emerge as the only pairwise strongly stable network. Let us recall that in an empty network, all firms are active in all markets. We then infer that the antitrust policy is a pro-competitive one.

As it is well known, in Cournot oligopolies with homogeneous goods, the social surplus (V) is increasing in the number of active firms on the market.

Accordingly, for a given α, the network configuration that maximizes V is one that involves more firms present on foreign markets, that is a network where firms have fewer connections among them. From, the Example 4, for $\alpha = 0.05$, the network configuration $\{2,1,1,1\}$ maximizes welfare.

From Proposition 6, as α increases, in particular, when $\alpha > \alpha^c$, the g^e is the only network that prevails over time. Therefore, in such a case, V would be the maximum.

Although $\alpha > \alpha^c$ may be the "advice" to give to the AA, it may not be the optimal antitrust policy, as the necessary costs to attain that enforcement level may outweigh its positive impact on the social surplus. That is, in order to know whether the AA has a net positive effect on social welfare, we must also consider the cost of enforcement.

Therefore, the net social welfare, W, depends on the network structure g (which depends, at last, on the particular level of α), as well as, on the cost of initiating a market-sharing agreement suit against a firm, $C(\alpha)$.

Therefore, if the AA were concerned about the optimal antitrust policy, then it would have to choose α such that it maximizes

$$W(g(\alpha), C) = V(g(\alpha)) - C(\alpha)$$

Unfortunately, the optimal antitrust policy is difficult to evaluate in our context because of the multiplicity in network configurations for each level of antitrust enforcement. In our network context, $g(\alpha)$ is not unique for each α. Moreover, a particular network g can emerge as being pairwise stable for different levels of α.

5 Concluding Remarks

We have characterized the stable collusive network that arises when firms form market-sharing agreements among themselves in a symmetric oligopolistic setting when an antitrust authority exists.

In this network framework, the incentives to participate in a collusive agreement are weakened by the AA because it reduces the net expected benefit from signing them. Under the presence of the AA, the expected penalties of forming illegal links appear, and they are positively related with the network configuration. This is because of two facts. First, firms, considering whether to sign an agreement, take into account the probability of being discovered rather than the probability of being inspected and the first probability positively depends on the number of agreements each firm has signed. Second, the fine imposed by the AA on a guilty firm is equal to its total profits, which depend on the number of active firms in its home market and also on the number of active firms in all foreign markets in which the guilty firm does not collude. Consequently, the penalty will be greater as the number of active firms in these markets is smaller, that is, as the number of links is larger.

We have shown that, the pairwise stable network can be decomposed into a set of isolated firms and complete components of different sizes. When the AA exists, however, we cannot define a unique lower bound on the size of complete components because it now depends on each network configuration and on probability of each of being inspected. In turn, this implies that, although the lower bound on the size of complete components may be greater, the set of isolated firms enlarges and, finally, more structures that are competitive can be sustained through bilateral agreements.

We have also shown that antitrust laws have a pro-competitive effect, as they give firms in large alliances more incentives to cut their agreements at once. Therefore, the empty network might arise as the only strongly stable network.

Although the optimal deterrence policy is beyond of the scope of the current article, an important policy implication of the present formulation is that the organization of the illegal behavior matters. That is, the analysis of the optimal deterrence of market-sharing agreements has to take into account the organizational structure of collusive firms. Furthermore, without considering the effects of the organizational structure, empirical studies may overestimate the contribution of efforts devoted to investigate and prosecute collusive agreements.[27]

In this article, we consider a relatively simple setting for analyzing the effect of the antitrust policy on the structure of criminal behavior. One can then diversify from here in many directions. One of them is to consider that the probability of inspection is sensitive to the network structure. This introduces some asymmetry among firms, which may then change the criminal network's configuration. Another extension to this article is to introduce a more complex but realistic context. A particular extension is how the internal structure of these conspiracies may affect their observable behavior, which, in turn, may throw some light on the optimal antitrust policy.

6 Appendix

Proof Result 1 As g is stable, when $g_{ij} = 1$ the next two conditions simultaneously hold:

$$\begin{aligned}
(1-\alpha)\,\pi(n_i) &\geq \pi(n_i+1) + \pi(n_j+1) + \alpha \sum_{k:g_{ik}=0} \pi(n_k) \\
(1-\alpha)\,\pi(n_j) &\geq \pi(n_j+1) + \pi(n_i+1) + \alpha \sum_{k:g_{jk}=0} \pi(n_k)
\end{aligned}$$

Given that the profit function is decreasing in n, the following are a pair of necessary conditions that must be satisfied for the above inequalities to hold:

$$\begin{aligned}
\pi(n_i) &> \pi(n_j+1) \\
\pi(n_j) &> \pi(n_i+1)
\end{aligned}$$

From the first inequality, $n_i < n_j + 1$, and from the second one, $n_j < n_i + 1$. Hence:

$$n_j - 1 < n_i < n_j + 1 \Leftrightarrow n_i = n_j$$

[27] Some empirical papers that estimate the deterrent effect of the policy are, among others, Buccirossi and Spagnolo,2005; Connor, 2006; Zimmerman and Connor, 2005.

That is,

$$n_i = n_j \equiv n \blacksquare$$

Proof Result 2

Part 1: If g is stable, then any component $g' \in g$ is complete. Suppose that g' is not complete. Then, there are three firms i, j, l in the component such that $g_{ij} = g_{jl} = 1$ and $g_{il} = 0$. Because g is stable, then by Result 1 $n_i = n_j \equiv n$ and $n_j = n_l \equiv n$, then $n_i = n_j = n_l \equiv n$. From the stability conditions, we are able to rewrite J_j^i as follows:

$$\underbrace{\frac{\pi(n)}{\pi(n+1)}}_{A} \geq \underbrace{\frac{2}{(1-\alpha)} + \frac{\alpha \sum\limits_{k:g_{ik}=0, i\neq k} \pi^i(n_k(g))}{(1-\alpha)\pi(n+1)}}_{B}$$

The same applies for J_i^j, J_l^j and J_j^l.

Given that $g_{il} = 0$, then one or both conditions hold for $h = i$ and/or $h = l$:

$$\underbrace{\frac{\pi(n-1)}{\pi(n)}}_{D} < \underbrace{\frac{2}{(1-\alpha)} + \frac{\alpha \sum\limits_{k:g_{hk}=0, h\neq k} \pi^i(n_k(g))}{(1-\alpha)\pi(n)}}_{E}$$

By log-convexity, we can establish that

$$A \leq D$$

From stability:

$$B \leq A \leq D < E$$

However, given that profits are decreasing functions, and given that the number of terms in $\sum\limits_{k:g_{ik}=0} \pi^i(n_k)$ in B and E are different, we can say that

$$B > E$$

This is a contradiction. Therefore g' must be a complete component.$\blacksquare$

Part 2: If g is stable, then the complete components must have different sizes. Take two firms i, j in component g' and a firm l in g''. Suppose, by contradiction, that $m(g') + 1 = m(g'') + 1$. Therefore, we have $n_i = n_j = n_l \equiv n$. The stability of g implies that $J_j^i \geq 0$ and $J_i^j \geq 0$. $J_j^i \geq 0$ can thus be written as

$$\underbrace{\frac{\pi(n)}{\pi(n+1)}}_{A} \geq \underbrace{\frac{2}{(1-\alpha)} + \frac{\alpha \sum_{k:g_{ik}=0, i\neq k} \pi^i(n_k(g))}{(1-\alpha)\pi(n+1)}}_{B}$$

(Similar expression for $J_i^j \geq 0$.)
For $h = i$ and/or $h = l$, the following condition holds:

$$\underbrace{\frac{\pi(n-1)}{\pi(n)}}_{D} < \underbrace{\frac{2}{(1-\alpha)} + \frac{\alpha \sum_{k:g_{hk}=0,\ h\neq k} \pi^i(n_k(g))}{(1-\alpha)\pi(n)}}_{E}$$

By log-convexity, we can establish that

$$A \leq D$$

From the stability conditions

$$B \leq A \leq D < E$$

However, given that profits are decreasing functions, and given that the number of terms in $\sum_{k:g_{ik}=0} \pi^i(n_k)$ in B and E are different, we can say that

$$B > E$$

Nevertheless, it is a contradiction with the assumption that profits are log-convex and with the stability of g.■

Proof Lemma 1 If $i \in g_h^*$ does not have incentives to cut a link with a firm inside its component, it is true that

$$\frac{\pi(N-m(g_h^*))}{\pi(N-m(g_h^*)+1)} > \frac{2}{(1-\alpha)} + \frac{\alpha\left[(m(g_l)+1)\pi(N-m(g_l)) + \sum_{k:g_{ik}=0}\pi(n_k)\right]}{(1-\alpha)\pi(N-m(g_h^*)+1)} \quad (9)$$

Assume by contradiction that $j \in g_l$ for $m(g_l) > m(g_h^*)$ has an incentive to cut a link with a firm inside its component. Then,

$$\frac{\pi(N-m(g_l))}{\pi(N-m(g_l)+1)} < \frac{2}{(1-\alpha)} + \frac{\alpha\left[(m(g_h^*)+1)\pi(N-m(g_h^*)) + \sum_{k:g_{jk}=0}\pi(n_k)\right]}{(1-\alpha)\pi(N-m(g_l)+1)} \quad (10)$$

when profits are decreasing in n, then RHS(9)>RHS(10). By the log-convexity assumption LHS(9)<LHS(10). Therefore, if i does not have an incentive to cut a link with a firm inside its component, LHS(9)> RHS(9), then LHS(10)>RHS(10), which contradicts (10).■

Proof Proposition 1 Result 1 and 2 and Lemma 1 provide the necessary conditions for stability. Let us consider the sufficiency part. Consider a network g that can be decomposed into a set of isolated firms and distinct complete components, g_1,...,g_L of different sizes $m(g_l) \neq m(g_{l'})$,$\forall l, l'$. Isolated players have no incentive to create a link with another isolated one. As long as a firm i, which belongs to the smallest component, does not have incentives to cut a link with a firm inside its component, then, by Lemma 3, no firm inside a component has incentives to cut a link. Additionally, given that $m(g_l) \neq m(g_{l'})$,$\forall l, l'$, there do not exist two firms belonging to different components that have an incentive to form an agreement between themselves.■

Proof Proposition 2 $\Rightarrow$ Consider a pairwise strong Nash equilibrium s^*. Given that any strongly pairwise stable network is pairwise stable, $g(s^*)$ can be decomposed into a set of isolated firms and complete components where no isolated firm wants to form a link with another isolated one and (9) holds. However, assume, by contradiction, that some component g_l does not satisfy the condition $(1-\alpha)^m \pi(N-m+1) \geq \pi(N)+(m-1)\pi(N-m+2)+\sum \pi(n_k)(1-(1-\alpha)^m)$ $\forall m = m(g_l)$. Then, s^* is not a Nash equilibrium, as any firm i in g_l has a profitable deviation by choosing $s_i' = \emptyset$.

$\Longleftarrow$ Assume that network g can be decomposed into a set of isolated firms and complete components of different sizes, where inequality (9) holds. Also assume that $(1-\alpha)^m \pi(N-m+1) \geq \pi(N) + (m-1)\pi(N-m+2) + \sum \pi(n_k)(1-(1-\alpha)^m)$ holds for all $m = m(g_l)$. We will show that the following strategies form a pairwise strong Nash equilibrium. For firm $i \in g_l$, it announces $s_i^* = \{j | j \in g_l, j \neq i\}$; however, if i is isolated, it announces $s_i^* = \emptyset$. Hence,

a) No isolated firm i has an incentive to create a link with another firm j, as $i \notin s_j^*$.

b) As $(1-\alpha)^m \pi(N-m+1) \geq \pi(N)+(m-1)\pi(N-m+2)+\sum \pi(n_k)(1-(1-\alpha)^m)$ holds for all $m = m(g_l)$, the firm has no incentive to destroy all of its m links. We must consider, however, the firm's incentives to cut a subset of them. Let us assume that it has an incentive to delete a strict subset of its links; hence, it chooses to delete h links because

$$(1-\alpha)^h \pi(N-m+1) < \pi(N-m+1+h)+h\pi(N-m+2)+\sum \pi(n_k)\left(1-(1-\alpha)^h\right)$$

Given that $h \geq 1$, then

$$\pi(N-m+1+h) + h\pi(N-m+2) \leq (h+1)\pi(N-m+2)$$

Because we are considering a strict subset of links, then $h < m-1$ and $h+1 < m-1$, and, as a result,

$$(h+1)\pi(N-m+2) < (m-1)\pi(N-m+2)$$

Therefore,

$$(1-\alpha)^m \pi(N-m+1) < (1-\alpha)^h \pi(N-m+1) < (m-1)\pi(N-m+2)$$

This contradicts our hypothesis.

c) No firm $i \in g_l$ has an incentive to create a link with firm $j \in g_{l'}$ as $i \notin s_j^*$. Moreover, as $m(g_l) \neq m(g_{l'})$ for all $l \neq l'$, no pair of firms $i \in g_l$ and $j \in g_{l'}$ has an incentive to create a new link between them.

d) As $(1-\alpha)^m \pi(N-m+1) \geq \pi(N) + (m-1)\pi(N-m+2) + \sum \pi(n_k)(1-(1-\alpha)^m)$ holds for all $m = m(g_l)$, when $m > 3$, no pair of firms have incentives to delete all their links or a subsets of their agreements and form a link between them. Let us assume, by contradiction, a pair of firms, $i \in m$ and $j \in m'$, has incentives to destroy all their m and m' links each and form a link between them. For firm i, this is

$$\begin{aligned} & (1-\alpha)^{m-2}\pi(N-m+1) \\ < \; & \pi(N-1) + (m-1)\pi(N-m+2) + (m'-1)\pi(N-m'+2) + \end{aligned}$$

$$+ \sum_{k \neq j \neq i, g_{ik=0},} \pi(n_k) - (1-\alpha)^{m-2}\left[\sum_{k \neq j \neq i, g_{ik=0},} \pi(n_k) + m'\pi(N-m'+1)\right] \tag{11}$$

Given that LHS(11)>LHS(8) and by straightforward computations, we can show that RHS(8)>RHS(11), when condition(8) holds then LHS(11)>RHS(11), which contradicts (11).■

Proof Proposition 3 $(\Longrightarrow)$ If g^c is pairwise stable, then

$$(1-\alpha)\pi(1) \geq 2\pi(2) \tag{12}$$

By rewriting the last condition, we get $\alpha \leq \alpha^c = 1 - \frac{2\pi(2)}{\pi(1)}$.

$(\Longleftarrow)$ If $\alpha \leq \alpha^c = 1 - \frac{2\pi(2)}{\pi(1)}$, then $(1-\alpha)\pi(1) \geq 2\pi(2)$. Therefore, g^c will be pairwise stable.■

Proof Proposition 4 Assume that $N \geq 3$.
$(\Longrightarrow)$ If g^e is pairwise stable then,

$$(1-\alpha)^2[\pi(N-1) + (N-2)\pi(N)] < \pi(N) + \pi(N) + (N-2)\pi(N) \tag{13}$$

and, by straightforward calculation,

$$\alpha > 1 - \left[\frac{N\pi (N)}{[\pi (N-1) + (N-2)\pi (N)]} \right]^{\frac{1}{2}} = \alpha_e (N)$$

($\Longleftarrow$) If $\alpha > \alpha_e (N)$, then (13) holds. Therefore, g^e is pairwise stable. ■

Proof Proposition 5 For simplicity, let us assume two complete components g_1 and g_2. For each firm $i \in g_1$, n_1 is the number of active firms in its market, and for each firm $j \in g_2$, n_2 is the number of active firms in its market.

Let us define $\alpha^* (n_i) := \frac{\pi(n_i-1)-2\pi(n_i)}{\pi(n_i-1)+\sum\limits_{k\neq j, gi=0} \pi(n_k)}$.

We are interested in knowing whether $\alpha^* (n_1) \lesseqgtr \alpha^* (n_2)$. That is,

$$\frac{\pi (n_1 - 1) - 2\pi (n_1)}{\pi (n_1 - 1) + (N - n_2 + 1)\pi (n_2)} \lesseqgtr \frac{\pi (n_2 - 1) - 2\pi (n_2)}{\pi (n_2 - 1) + (N - n_1 + 1)\pi (n_1)}$$

By solving the last expression, we get

$$(N - n_1 + 1)\pi (n_1)\pi (n_1 - 1) - 2\pi (n_1)\pi (n_2 - 1) - 2(N - n_1 + 1)[\pi (n_1)]^2 \lesseqgtr$$
$$(N - n_2 + 1)\pi (n_2)\pi (n_2 - 1) - 2\pi (n_2)\pi (n_1 - 1) - 2(N - n_2 + 1)[\pi (n_2)]^2$$

In order to decide the sense of the inequality, we rearrange the above expression into the following two parts:

$$(N - n_1 + 1)\pi (n_1)[\pi (n_1 - 1) - 2\pi (n_1)] \lesseqgtr (N - n_2 + 1)\pi (n_2)[\pi (n_2 - 1) - 2\pi (n_2)]$$

$$\pi (n_1)\pi (n_2 - 1) \lesseqgtr \pi (n_2)\pi (n_1 - 1)$$

If $n_1 > n_2$, then (i) $(N - n_1 + 1) < (N - n_2 + 1)$; (ii) by Property 1, $\pi (n_1) < \pi (n_2)$; (iii) by Property 2, $[\pi (n_1 - 1) - 2\pi (n_1)] < [\pi (n_2 - 1) - 2\pi (n_2)]$.

Therefore,

$$(N - n_1 + 1)\pi (n_1)[\pi (n_1 - 1) - 2\pi (n_1)] < (N - n_2 + 1)\pi (n_2)[\pi (n_2 - 1) - 2\pi (n_2)] \tag{14}$$

Additionally, if $n_1 > n_2$, then, by the log-convexity assumption, $\frac{\pi(n_2-1)}{\pi(n_2)} > \frac{\pi(n_1-1)}{\pi(n_1)}$

Hence,

$$\pi (n_1)\pi (n_2 - 1) > \pi (n_2)\pi (n_1 - 1) \tag{15}$$

Therefore, if, $n_1 > n_2$, by (14) and (15), then

$$\alpha^* (n_1) < \alpha^* (n_2) \blacksquare$$

Proof Proposition 6 By Claim 1, we know that, at $\alpha = \alpha^c$, $g^e$and g^c are pairwise stable.

Now, we must check, for $\alpha = \alpha^c$, whether a firm i has incentive to form an additional agreement when $n \neq 1$ and $n \neq N$.

Therefore, we must verify whether $J_j^i \lesseqgtr 0$, that is,

$$\pi (n-1) - 2\pi (n) \lesseqgtr \alpha \left(\pi (n-1) + \sum_{k \neq j, g_{ki}=0} \pi (n_k) \right)$$

At $\alpha = \alpha^c$, the above expression is

$$\pi (n-1) - 2\pi (n) \lesseqgtr \left[1 - \frac{2\pi (2)}{\pi (1)} \right] \left(\pi (n-1) + \sum_{k \neq j, g_{ki}=0} \pi (n_k) \right)$$

After some calculations, we obtain

$$2 \left[\pi (n-1) \pi (2) - \pi (n) \pi (1) \right] \lesseqgtr \sum_{k \neq j, g_{ki}=0} \pi (n_k) \left[\pi (1) - 2\pi (2) \right]$$

From Property 2, we know that, $\pi (1) - 2\pi (2) > 0$, and given the log-convexity assumption, $[\pi (n-1) \pi (2) - \pi (n) \pi (1)] < 0$. Therefore, at $\alpha = \alpha^c$,

$$J_j^i < 0 \blacksquare$$

Proof Proposition 7 The partial derivative of (8) respect to α is

$$-(m+1) \left[\pi (N-m+1) + \sum \pi (n_k) \right] (1-\alpha)^m \tag{16}$$

That is, as α increases, the incentive to maintain links decreases.

Now, we must check whether (16) is larger for firms in large components. Without a loss of generality, assume that there are two components whose sizes are $m_1 + 1$ and $m_2 + 1$ respectively, such that $m_1 > m_2$. After some computations, we can verify that, for a sufficiently high m, the following holds:

$$-(m_1+1) \left[\pi (N-m_1+1) + m_2 \pi (N-m_2+1) \right] (1-\alpha)^{m_1} <$$
$$-(m_2+1) \left[\pi (N-m_2+1) + m_1 \pi (N-m_1+1) \right] (1-\alpha)^{m_2} \blacksquare$$

References

Aumann, R. and Myerson, R. 1988. "Endogenous formation links between players and coalitions: an application of the Shapley value." In Roth, A. *The Shapley Value.* Cambridge University Press, Cambridge, United Kingdom, 1988.

Ballester, C., Calvó-Armengol, A. and Zenou, Y. "Who's who in networks. Wanted: the key player." *Econometrica*, Vol. 74 (2006), pp. 1403-1417.

Becker, G. "Crime and Punishment: An economic approach." *Journal of Political Economy*, Vol. 76 (1968), pp. 169-217.

Belleflamme, P. and Bloch, F. "Market-sharing agreements and collusive networks." *International Economic Review*, Vol. 45 (2004), pp. 387-411.

Besanko, D. and Spulber, D. "Antitrust enforcement under asymmetric information." *The Economic Journal*, Vol. 99 (1989), pp. 408-425.

———— and ———— "Are Treble Damages Neutral? Sequential Equilibrium and Private Antitrust Enforcement." *American Economic Review*, Vol. 80 (1990), pp. 870-887.

Block, M. et al "The deterrent effect of antitrust enforcement." *Journal of Political Economy*, Vol. 89 (1981), pp. 429-445.

Calvó-Armengol, A. and Zenou, Y., "Social networks and crime decisions: the role of social structure in facilitating delinquent behavior." *International Economic Review*, Vol. 45 (2004), pp. 939-958.

Chen, Z. and Rey,P., "On the design of Leniency Programs", Working Paper, N^o. 452, IDEI, University of Toulouse, 2007.

D'Aspremont, C et al "On the stability of Collusive Price Leadership". *Canadian Journal of Economics*, Vol. 16 (1983), pp. 17-25.

Deneckere, R. and Whinston, M. "Multimarket contact and collusive behavior." *Rand Journal of Economics*, Vol. 21 (1990), pp. 1-26.

Dutta, B. and Mutuswami, S. "Stable Networks." *Journal of Economic Theory*, Vol. 76 (1997), pp. 322-344.

Goyal, S., "Sustainable Communications Networks." Discussion Paper TI 93-250, Tinbergen Institute, 1983.

———, and Joshi, S. "Networks of Collaboration in Oligopoly." *Games and Economic Behavior*, Vol. 43 (2003), pp. 57-85.

———, and Moraga, J. "R&D Networks." *Rand Journal of Economics*, Vol. 32 (2001), pp. 686-707.

Harrington, Jr.,J.E. "Some implications of antitrust laws of cartel pricing." *Economics Letters*, Vol. 79 (2003), pp. 377-383.

Harrington, Jr.,J.E. "Cartel Pricing Dynamics in the Presence of an Antitrust Authority." *Rand Journal of Economics*, Vol. 35 (2004), pp. 651-673

Harrington, Jr.,J.E. "Optimal Cartel Pricing in the. Presence of an Antitrust Authority." *International Economic Review*, Vol. 46 (2005), pp. 145-169

Jackson, M. and A. van den Nouweland "Strongly stable networks." *Games and Economic Behavior*, Vol. 51 (2005), pp. 420-444.

———— and Wolinsky, A. "A strategic model of social and economic networks." *Journal of Economic Theory*, Vol. 71 (1996), pp. 44-74.

LaCasse, C. "Bid Rigging and the Threat of Government Prosecution." *Rand Journal of Economics*, Vol. 26 (1995), pp. 398-417

Motta, M., *Competition Policy: Theory and Practice.* Cambridge University Press, 2004.

————and Polo, M. "Leniency Programs and Cartel Prosecution." *International Journal of Industrial Organization*, Vol. 21 (2003), pp. 347-379.

Myerson, R.B., *Game theory: Analysis of conflict.* Cambridge, MA: Harvard University Press, 1991.

Mitchell Polinsky, A. and Shavell, S. "The economic theory of public enforcement of law." *Journal of Economic Literature*, Vol. 38 (2000), pp. 45-76.

Polo, M., "The Optimal Prudential Deterrence of Price Fixing Agreements". Working Paper N^o 120, IGER, 1997

Rey, P., "Towards a Theory of Competition Policy." Working Paper N^o. 121, IDEI, University of Toulouse, 2002.

Roldán, F. "Collusive Networks in Market Sharing Agreements in the Presence of an Antitrust Authority", Working Paper. N^o 08-50-24, Department of Economics, Universidad Carlos III de Madrid, 2008.

Scherer, F. and Ross, D., *Industrial Market Structure and Economic Performance,* 3rd. Ed. Boston: Houghton-Mifflin, 1990.

Stigler, G. "Monopoly and Oligopoly by Merger." *American Economic Review*, Vol. 40 (1950), pp. 23-34.

Vives, X., *Oligopoly Pricing: Old Ideas and New Tools.* The MIT Press Editor, 1999.