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Abstract

This paper corrects some of the equations of Farber, Gillet and Szafarz (2006). The WACC is a discount rate widely used in corporate finance. However, correctly calculating the WACC involves properly calculating the value of tax shields, and the value of tax shields depends on the company's debt policy. Many authors (e.g. Inselbag and Kaufold (1997), Booth (2002), Cooper and Nyborg (2006), Farber, Gillet and Szafarz (2006)) have stated that debt policy can only be implemented by maintaining a fixed market-value debt ratio (Miles-Ezzell's assumption) or a fixed dollar amount of debt (Modigliani-Miller's assumption).

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The value of tax shields (VTS) defines the increase in the company's value as a result of the tax saving obtained by paying interest. However, there is no consensus in the existing literature regarding the correct way to calculate the VTS. Modigliani and Miller (1963), Myers (1974), Luehrman (1997), Brealey and Myers (2000) and Damodaran (2006) propose discounting the tax savings arising from interest payments on debt at the cost of debt (r_D), whereas Harris and Pringle (1985) and Ruback (1995, 2002) propose discounting these tax savings at the cost of capital for the unlevered firm (r_A). Miles and Ezzell (1985) propose discounting these tax savings at the cost of debt in the first year and at Ku in subsequent years.

Farber, Gillet and Szafarz (2006) start their paper with the value of the levered firm:

$$E + D = Vu + VTS \tag{1}$$

where E is the value of equity, D is the value of debt, Vu is the value of unlevered equity and VTS is the value of tax shields. From equation (1), we may derive equation (8) of Farber, Gillet and Szafarz (2006):

$$E\, r_E + D\, r_D = Vu\, r_A + VTS\, r_{TS} \tag{2}$$

where r_E, r_D, r_A and r_{TS} are the required return on anticipated cash flow for equity, debt, assets (free cash flow) and tax shields. This equation is correct, but what is incorrect is the assumption made by Farber, Gillet and Szafarz (2006) that required return is always constant over time. Specifically, they state that r_{TS} can be r_D (as Modigliani-Miller do) or r_A (as Harris-Pringle do). These two scenarios correspond to two different financing strategies: the first one is valid for a company that has a preset amount of debt and the second one should be valid for a company that has a fixed leverage ratio in market-value terms [D = L (D + E)]. However, as Miles and Ezzell (1985) and Arzac and Glosten (2005) have shown, the required return for the tax shield (r_{TS}) of a company that has a fixed leverage ratio in market-value terms is r_D for the tax shields of the first period and r_A thereafter. It is not possible to derive a debt policy for which the appropriate discount rate for tax shields is r_A in all periods. $D_t = L \cdot (D_t + E_t)$ implies that D_t is also proportional to FCF_t. The correct Miles and Ezzell (1985) formula for the VTS of a perpetuity growing at rate g is:

$$VTS^{ME} = \frac{D r_D T}{(r_A - g)} \frac{(1 + r_A)}{(1 + r_D)} \tag{3}$$

Formula (3) is identical to formula (21) of Miles and Ezzell (1985), formula (13) of Arzac and Glosten (2005) and formula (7) of Lewellen and Emery (1986). However, Farber, Gillet

and zafarz (2006) and Harris and Pringle (1985) present a formula that does not correspond to the ME assumption:

$$VTS^{HP} = \frac{Dr_D\,T}{(r_A - g)} \tag{4}$$

If debt is adjusted continuously, not only at the end of the period, then the ME formula (3) changes to

$$VTS = D\,\rho\,T/(\kappa - \gamma) \tag{5}$$

where $\rho = \ln(1+ r_D)$, $\gamma = \ln(1+g)$, and $\kappa = \ln(1+ r_A)$. Perhaps formula (5) induces Farber *et al.* (2006) and Harris and Pringle (1985) to use (4) as the expression for the value of tax shields when the company maintains a constant market-value leverage ratio (but then r_D, g and r_A should also be expressed in continuous time). (4) is incorrect for discrete time: (3) is the correct formula.

As a result of this error, the subsequent equations of Farber *et al.* (2006) should be modified:

Equations (14) and (28) should be:

$$r_E = r_A + \frac{D}{E}(r_A - r_D)\frac{1+R_F(1-T)}{1+R_F} \text{ , instead of } r_E = r_A + \frac{D}{E}(r_A - r_D) \tag{6}$$

Equations (25) and (29) should be:

$$WACC = r_A - L\frac{r_D T(1+r_A)}{(1+r_D)} \text{ , instead of } WACC = r_A - Lr_D T \tag{7}$$

Required return on equity and WACC

For perpetuities with a constant growth rate (g), the relationship between anticipated values in t=1 of free cash flow (FCF) and equity cash flow (ECF) is:

$$ECF_0(1+g) = FCF_0(1+g) - D_0\,r_D\,(1-T) + g\,D_0 \tag{8}$$

The value of equity today (E) is equal to the present value of the anticipated equity cash flow. If r_E is the average appropriate discount rate for the anticipated equity cash flow, then $E = ECF_0(1+g) / (r_E - g)$, and equation (8) is equivalent to:

$$E\,r_E = Vu\,r_A - D\,r_D + VTS\,g + D\,r_D\,T \tag{9}$$

And the general equation for r_E is:

$$r_E = r_A + \frac{D}{E}[r_A - r_D(1-T)] - \frac{VTS}{E}(r_A - g) \tag{10}$$

(10) is equivalent to equation (10) of Farber *et al.* (2006) because $VTS = D\,r_D\,T / (r_{TS} - g)$.

The WACC is the appropriate discount rate for anticipated free cash flow, in which $D_0+E_0= FCF_0(1+g) / (WACC-g)$. The equation that links the WACC to the VTS is (11):

$$WACC = r_A\left(1-\frac{VTS}{D+E}\right)+\frac{gVTS}{D+E} \qquad (11)$$

(10) is equivalent to equation (18) of Farber *et al.* (2006) because $VTS = D\ r_D\ T / (r_{TS} - g)$.

Conclusions

The WACC is a discount rate widely used in corporate finance. However, correctly calculating the WACC involves properly calculating the value of tax shields, and the value of tax shields depends on the company's debt policy. When the debt level is fixed, the Modigliani-Miller approach applies and tax shields should be discounted at the required return on debt. If the leverage ratio is fixed at market value, then the Miles-Ezzell approach applies. Other debt policies should be explored. For example, Fernandez (2006) develops valuation formulae for the situation in which the leverage ratio is fixed at book value and argues that it is more realistic to assume that a company will maintain a fixed book-value leverage ratio than to assume, as Miles-Ezzell do, that the company will maintain a fixed market-value leverage ratio because this will make the company more valuable, and because it is easier for non quoted companies to follow.

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