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FINANCIAL LITERATURE ABOUT DISCOUNTED
CASH FLOW VALUATION

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FINANCIAL LITERATURE ABOUT DISCOUNTED CASH FLOW VALUATION

Abstract

There is a wealth of literature about discounted cash flow valuation. In this paper, we will discuss the most important papers, highlighting those that propose different expressions for the value of the tax shield (VTS).

The discrepancies between the various theories on the valuation of a company's equity using discounted cash flows originate in the calculation of the value of the tax shield (VTS). This paper illustrates and analyzes 7 different theories and presents a new interpretation of the theories.

JEL classification: G12; G31; G32

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FINANCIAL LITERATURE ABOUT DISCOUNTED CASH FLOW VALUATION

There is a wealth of literature about discounted cash flow valuation. In this paper, we discuss the most important papers¹, highlighting those that propose different expressions for the value of the tax shield (VTS).

The discrepancies between the various theories on the valuation of a company's equity using discounted cash flows originate in the calculation of the value of the tax shield (VTS). This paper illustrates and analyzes 7 different theories on the calculation of the VTS: Fernández (2004), Myers (1974), Miller (1977), Miles & Ezzell (1980), Harris & Pringle (1985), Ruback (1995), Damodaran (1994), and the practitioners' method. We show that Myers' method (1974) gives inconsistent results for growing companies. This paper also presents a new interpretation of the theories: it is argued that the difference between the company's value given by Fernández (2004) (zero failure costs) and the company's value given by these theories is the leverage cost. When analyzing the results obtained by the different theories, it is advisable to remember that the VTS is not exactly the present value of the tax shield discounted at a certain rate but the difference between two present values: the present value of the taxes paid by the unlevered company less the present value of the taxes paid by the levered company. The risk of the taxes paid by the unlevered company is smaller than the risk of the taxes paid by the levered company.

1. A brief review of the most significant papers

Gordon & E. Shapiro (1956) showed that the present value of a flow F growing at the rate g , when discounted at the rate K , is: $PV_0 = F_1 / (K-g)$

Modigliani and Miller (1958) studied the effect of leverage on the firm's value. Their proposition 1 (1958, formula 3) states that, in the absence of taxes, the firm's value is independent of its debt, i.e., $E + D = Vu$, if $T = 0$. E is the equity value, D is the debt value, Vu is the value of the unlevered company and T is the tax rate.

¹ It is a good idea to know what the experts on the subject have said in order not to make the mistake that Seneca warned us against: "He who is his own master becomes disciple to an ass".

In the presence of taxes, their second proposition (**1963**, formula 12.c) states that the required return on equity flows (K_e) increases at a rate that is directly proportional to the debt to equity ratio (D/E) at market value:

$$[1] K_e = K_u + (D/E) (1-T) (K_u - K_d)$$

In the presence of taxes and for the case of perpetuities, their first proposition is transformed into (1963, formula 3):

$$[2] E^0 + D^0 = V_u + D T$$

DT is the value of the tax shield (VTS) for perpetuities. But it is important to note that they arrive at the value of the tax shield (VTS) by discounting the present value of the tax savings due to interest payments of a risk-free debt ($T D R_F$) at the risk-free rate (R_F).

They also state in their paper (1963, formula 33.c) that, in an investment that can be financed totally by debt, the required return on the debt must be equal to the required return on the asset flows: if $D / (D+E) = 100\%$, $K_d = K_u$.

The purpose of Modigliani and Miller was to illustrate the tax impact of debt on value. They never addressed the issue of the riskiness of the taxes and only dealt with perpetuities. If we relax the no-growth assumption, then new formulas are needed.

In the case of dividends, Modigliani and Miller said that they were irrelevant if the taxes on dividends and capital gains were the same. Given equal taxes, the shareholder would have no preference between receiving dividends or selling shares.

Modigliani & Miller (1963) give a number of valuation formulas that we shall use in this paper:

Their formula (31.c) is: $WACC = K_u [1 - T D / (E+D)]$.

Their formula (11.c) is: $WACC_{BT} = K_u - D T (K_u - K_d) / (E+D)$.

However, in their last equation, Modigliani & Miller (1963) propose calculating the company's target financial structure [$D / (D+E)$] using book values for D and E , instead of market values. This is obviously incorrect.

Myers (1974) was responsible for introducing the APV (*adjusted present value*). According to Myers, the value of the levered company is equal to the value of the debt-free company (V_u) plus the present value of the tax shield due to the payment of interest (VTS). Myers proposes that the VTS be calculated as follows:

$$[3] VTS = PV[K_d; T D K_d]$$

The argument is that the risk of the tax saving arising from the use of debt is the same as the risk of the debt. **Luehrman (1997)** recommends that companies be valued using the Adjusted Present Value and calculates the VTS in the same way as Myers. The company's value is: $APV = E + D = V_u + VTS = PV[K_u; FCF] + PV[K_d; T D K_d]$.

Benninga and Sarig (1997) claim that if there are personal taxes, the tax benefits of the debt should be discounted with after-personal-tax discount rates. According to them,

$$VTS = PV [K_d (1 - T_{PD}); D K_d [(1 - T_{PD}) - (1 - T) (1 - T_{PA})]]$$

Corporate income tax is T , the personal tax rate on shares is T_{PA} and the personal tax rate on debt is T_{PD} . Note that if $T_{PA} = T_{PD}$, then Benninga and Sarig's formula becomes [3].

Arditti & Levy (1977) suggest that the company's value be calculated by discounting the *capital cash flows* (equity cash flow plus debt cash flow), instead of the *free cash flow*. The *capital cash flows* (CCF) must be discounted at the $WACC_{BT}$ (WACC before tax). It is readily shown that:

$D + E = PV[WACC; FCF] = PV[WACC_{BT}; CCF]$, where $WACC_{BT}$ is:

$$[4] \quad WACC_{BT_t} = K_e \frac{E_{t-1}}{E_{t-1} + D_{t-1}} + K_d \frac{D_{t-1}}{E_{t-1} + D_{t-1}}$$

Arditti & Levy's paper (1977) suffers from one basic problem: they calculate the weights of debt ($D / [E+D]$) and equity ($E / [E+D]$) at book value instead of market value. Hence their statement (p. 28) that the company's value obtained by discounting the FCF is different from that obtained by discounting the CCF.

Miller (1977) argues that while there is an optimal debt structure for companies as a whole, such a structure does not exist for each company. Miller argues that due to the clientele effect, debt does not add any value to the company. Consequently, according to Miller, $E+D = V_u$.

He also introduces personal income tax as well as corporate income tax. The tax rate for the company is T , the personal tax rate on shares is T_{PA} and the personal tax rate on debt is T_{PD} . According to Miller, for a perpetuity, the value of the debt-free company after personal income tax is $V_u = FCF (1 - T_{PA}) / K_u$. If the company has debt with a nominal value N , its value is: $D = N K_d (1 - T_{PD}) / K_d$.

Miller says that the value created by debt, in the case of perpetuities, is:

$$D [1 - (1-T) (1 - T_{PA}) / (1 - T_{PD})]$$

But he goes on to say (see p. 268) that any attempt by a company to increase its value by increasing its debt would be incompatible with market balance. The increased debt would generate changes in the required returns to debt and equity and in the shares' owners, with the result that the company's value will be independent of debt.

Miller also says that if $T_{PA} = 0$, the aggregate debt supply must be such that it offers an interest $R_0 / (1-T)$, where R_0 is the rate paid by tax-free institutions.

Miller & Scholes (1978) show that, even if the income tax rate is greater than the capital gains tax rate, many investors will not pay more than the capital gains tax rate charged on dividends. They conclude that investors will have no preference between receiving dividends or realizing capital gains if the company buys back shares. According to these authors, the company's value will not depend on its dividend policy, not even in the presence of corporate and personal income tax.

DeAngelo & Masulis (1980) expand on Miller's work. Considering that the marginal tax rate is different for different companies, they predict that companies will use less debt the greater their possibilities for reducing tax by other means: depreciation, deduction of investments...

Miles & Ezzell (1980) maintain that the APV and the WACC give different values: “unless debt and, consequently, K_e are exogenous (they do not depend on the company’s value at any given time), the traditional WACC is not appropriate for valuing companies”. According to them, a company that wishes to maintain a constant D/E ratio must not be valued in the same way as a company that has a preset amount of debt. Specifically, formula [20] in their paper states that for a company with a fixed target debt ratio $[D/(D+E)]$, the *free cash flow* (FCF) must be discounted at the rate:

$$\text{WACC} = K_u - [D / (E+D)] [K_d T (1+K_u) / (1+K_d)]$$

They arrive at this formula from their formula [11], which, for a growing perpetuity, is:

$$E_{t-1} + D_{t-1} = \text{FCF}_t / (K_u - g) + K_d T D_{t-1} / (K_u - g)$$

They say that the correct rate at which the tax saving due to debt ($K_d T D_{t-1}$) must be discounted is K_d for the first year’s tax saving and K_u for the following years’ tax savings. The expression of K_e is their formula [22]: $K_e = K_u + D (K_u - K_d) [1 + K_d (1-T)] / [(1+K_d) E]$

Miles & Ezzell (1985) show in their formula [27] that the relationship between levered beta and asset beta (assuming that the debt is risk-free and the debt’s beta is zero) is

$$\beta_L = \beta_u + D \beta_u [1 - T R_f / (1 + R_f)] / E$$

Chambers, Harris & Pringle (1982) compare four discounted cash flow valuation methods: the equity cash flow (ECF) at the rate K_e (required return to equity); the *free cash flow* (FCF) at the WACC (weighted average cost of capital); the *capital cash flow* (CCF) at the WACC_{BT} (weighted average cost of capital before tax); and Myers’ APV. They say that the first three methods give the same value if debt is constant, but different values if it is not constant. They also say that the APV only gives the same result as the other three methods in two cases: in companies with only one period, and in no-growth perpetuities. The reason for their results is an error: they calculate the debt ratio ($D/[D+E]$) using book values instead of market values. Their Exhibit 3 is proof of this: it is impossible for the WACC and K_e to be constant. If $K_e = 11.2\%$, as they propose, the correct WACC is 6.819% in the first year (instead of their 5.81%) and increases in the following years; and the correct WACC_{BT} is 7.738 % in the first year (instead of their 6.94%) and increases in following years. When the debt ratio ($D/[D+E]$) is calculated using market values, all three procedures give the same value.

Harris and Pringle (1985) propose that the present value of the tax saving due to the payment of interest (VTS) should be calculated by discounting the tax saving due to the debt ($K_d T D$) at the rate K_u :

$$[5] \text{ VTS} = \text{PV} [K_u; D K_d T]$$

They also propose in their formula [3] that $\text{WACC}_{\text{BT}} = K_u$ and, therefore, their expression for the WACC is:

$$[6] \text{ WACC} = K_u - D K_d T / (D + E)$$

Harris and Pringle (1985) say “the MM position is considered too extreme by some because it implies that interest tax shields are no more risky than the interest payments themselves. The Miller position is too extreme for some because it implies that debt cannot

benefit the firm at all. Thus, if the truth about the value of tax shields lies somewhere between the MM and Miller positions, a supporter of either Harris and Pringle or Miles and Ezzell can take comfort in the fact that both produce a result for unlevered returns between those of MM and Miller. A virtue of Harris and Pringle compared to Miles and Ezzell is its simplicity and straightforward intuitive explanation.”

Ruback (1995) assumes in his formula [2.6] that $\beta_L = \beta_U (D+E)/E - \beta_D D/E$. With this assumption, he arrives at formulas identical to those of Harris-Pringle (1985).

Kaplan and Ruback (1995) also calculate the VTS “discounting interest tax shields at the discount rate for an all-equity firm”.

Tham and Vélez-Pareja (2001), following an arbitrage argument, also claim that the appropriate discount rate for the tax shield is K_u , the return to unlevered equity. We will see later on that this theory provides inconsistent results.

Lewellen and Emery (1986) propose three alternative ways to calculate the VTS. They claim that the most logically consistent is the method proposed by Miles and Ezzell. But one method, which they label Modigliani-Miller, assumes the calculation (see their equation 15) of the VTS as: $PV[K_u; DTK_u]$. As will be discussed later in this paper, this is the only method that provides logically consistent values in a world without cost of leverage.

Taggart (1991) gives a good summary of valuation formulas with and without personal income tax. He proposes that Miles & Ezzell’s (1980) formulas should be used when the company adjusts to its target debt ratio once a year and Harris & Pringle’s (1985) formulas when the company continuously adjusts to its target debt ratio.

Damodaran (1994) argues² that if the business’s full risk is borne by the equity, then the formula that relates levered beta (β_L) to asset beta (β_U) is: $\beta_L = \beta_U + (D/E) \beta_U (1 - T)$. This expression is obtained from the relationship between levered beta, asset beta, and debt beta according to Fernández (2004)³, eliminating the debt beta. It is important to realize that eliminating the debt beta is not the same as assuming it is zero, as Damodaran says. If the debt beta were zero, the required return to debt should be the risk-free rate. The purpose of eliminating the debt beta is to obtain a higher levered beta (and a higher K_e and a lower equity value) than that given by Fernández (2004), which is equivalent to introducing leverage costs in the valuation

Another way of relating the levered beta to the asset beta is the following: $\beta_L = \beta_U (E+D)/E$. We will call this formula the **practitioners’ formula**, as it is a formula commonly used by consultants and investment banks⁴. Obviously, according to this formula, assuming that β_U is the same, a higher beta (higher leverage costs) is obtained than according to Fernández (2004) and Damodaran (1994).

Inselbag and Kaufold (1997) argue that if the firm targets the dollar values of debt outstanding, the VTS is given by the Myers formula. However, if the firm targets a constant debt/value ratio, the VTS is given by Miles and Ezzell’s formula⁵. The authors use the

² See page 31 of his book *Damodaran on valuation*. This expression of levered beta appears in many books and is frequently used by consultants and investment banks.

³ The relationship between levered beta, asset beta and debt beta, according to Fernández’s (2004) theory, is: $\beta_L = \beta_U + (D/E) (\beta_U - \beta_D) (1 - T)$. This relationship may also be obtained from Modigliani-Miller (1963) for perpetuities.

⁴ Two of the many places where it can be found are: Ruback (1995), p. 5; and Ruback (1989), p. 2.

⁵ Copeland (2000) suggest only this paper as additional reading on APV (p. 483).

example of a company, Media Inc., with two alternative financing strategies: first, setting the planned quantity of debt, and second, setting the debt ratio.

According to them, the present value of the tax shield due to the payment of interest (VTS) is greater if the company sets the planned quantity of debt than if it sets the debt ratio. We do not agree with this for two reasons. The first is that we do not see any companies firing their COO or CFO because they propose a target debt ratio (instead of fixing the quantity of debt). The second is that, as we have already said, the VTS is the difference between two present values: that of taxes in the unlevered company and that of taxes in the levered company. Inselbag and Kaufold argue that having a target debt ratio is riskier than setting the quantity of debt. If this were to be so, the present value of the taxes to be paid in the levered company should be greater in the company that sets the quantity of debt and, consequently, the VTS would be less, which is exactly the opposite of what they propose.

Copeland, Koller and Murrin (2000)⁶ treat the Adjusted Present Value in their Appendix A. They only mention perpetuities and propose only two ways of calculating the VTS: Harris and Pringle (1985) and Myers (1974). They conclude “*we leave it to the reader’s judgment to decide which approach best fits his or her situation*”. They also claim that “the finance literature does not provide a clear answer about which discount rate for the tax benefit of interest is theoretically correct”.

Fernández (2001) shows that the discounted value of tax shields is the difference between the present values of two different cash flows, each with its own risk: the present value of taxes for the unlevered company and the present value of taxes for the levered company. This implies as a first guideline that, for the particular case of a perpetuity and a world without costs of leverage, the discounted value of tax shields is equal to the tax rate times the value of debt (i.e. Fernández (2004), Myers and Modigliani-Miller). The discounted value of tax shields can be lower, when costs of leverage exist. In that case, it is shown that, since the existence of leverage costs is independent of taxes, a second guideline for the appropriateness of the valuation method should be that the discounted value of tax shields when there are no taxes is negative.

Twenty-three valuation theories proposed in the literature to estimate the present discounted value of tax shields are analyzed according to their performance relative to the proposed guidelines. By analyzing perpetuities, the author is able to eliminate 8 theories that not only do not provide us with a value of the tax shield of DT (as the candidates for a world without cost of leverage should), but also fail to provide us with a negative VTS when there are no taxes (as the candidates for a world with leverage cost should). The 8 candidates eliminated due to a lack of consistent results include Harris-Pringle (1985) or Ruback (1995), Miles-Ezzell (1980), and Miller (1977).

By analyzing constant growth companies, the author is able to see that there is only one theory that provides consistent results in a world without leverage cost. In accordance with this theory, the VTS is the present value of $DTKu$ discounted at the unlevered cost of equity (Ku). It is not the interest tax shield that is discounted.

The author finds three theories that provide consistent results in a world with leverage cost: Fernández (2001)⁷, Damodaran (1994) and Practitioners. Only Fernández (2001) is fully applicable, while the other two are applicable up to a certain point. The differences among the theories can be attributed to the implied leverage cost in each of them.

⁶ See p. 477.

⁷ According to Fernández (2001), $VTS = PV[Ku; D(KuT + R_F - Kd)]$.

Following an empirical approach, **Graham (2000)** estimates value creation due to debt at 9.7% of the company's value. If personal income tax is included, value creation is reduced to 4.3% of the company's value. The author concludes by saying "I suspect that many debt-conservative firms, if they objectively consider the issue, will reach the conclusion that they should use more debt".

2. Main formulas in the most significant papers

2.1. Different expressions of the Value of the tax shield and of the required return to equity

Table 1 contains the 8 most important theories. For each theory, the table contains the formula for calculating the VTS and the equation that relates the required return to equity, K_e , with the required return to assets (or required return to unlevered equity), K_u .

Table 1. Competing theories for calculating the value of the tax shields

	Theories	VTS	K_e
1	Fernández (2004)	$PV[Ku; DTKu]$	$K_e = K_u + \frac{D(1-T)}{E}(K_u - K_d)$
2	Damodaran (1994)	$PV[Ku; DTKu - D(K_d - R_f)(1-T)]$	$K_e = K_u + \frac{D(1-T)}{E}(K_u - R_f)$
3	Practitioners	$PV[Ku; T D K_d - D(K_d - R_f)]$	$K_e = K_u + \frac{D}{E}(K_u - R_f)$
4	Harris-Pringle (1985), Ruback (1995)	$PV[Ku; T D K_d]$	$K_e = K_u + \frac{D}{E}(K_u - K_d)$
5	Myers (1974)	$PV[K_d; T D K_d]$	$K_e = K_u + \frac{D - DVTS}{E}(K_u - K_d)$
6	Miles-Ezzell (1980)	$PV[Ku; T D K_d] (1+K_u)/(1+K_d)$	$K_e = K_u + \frac{D}{E}(K_u - K_d) \left[1 - \frac{T K_d}{1+K_d} \right]$
7	Miller (1977)	0	$K_e = K_u + \frac{D}{E}[K_u - K_d(1-T)]$
8	Fernández (2001)	$PV[Ku; D(K_u T + R_f - K_d)]$	$K_e = K_u + \frac{D}{E}[K_u(1-T) + K_d T - R_f]$

PV = Present value; T = Corporate tax rate; K_u = Cost of unlevered equity (required return of unlevered equity); K_e = Cost of levered equity (required return of levered equity); K_d = Required return of debt = Cost of debt; D = Value of debt; E = Value of equity; R_f = Risk free rate; WACC = Weighted average cost of capital;

According to the Fernández (2004) theory, the VTS is the present value of $DTKu$ (not the interest tax shield) discounted at the unlevered cost of equity (K_u). This theory implies that the relationship between the leveraged beta and the unlevered beta is

$$[7] \quad \beta_L = \beta_u + \frac{D(1-T)}{E}(\beta_u - \beta_d)$$

The second theory is that of Damodaran (1994). Although Damodaran does not mention what should be the discounted value of the tax shield, his formula relating the levered beta with the asset beta

$$[8] \quad \beta_L = \beta_u + \frac{D(1-T)}{E}\beta_u \quad \text{implies that VTS} = PV[Ku; DTKu - D(K_d - R_f)(1-T)]$$

It is important to notice that formula [8] is exactly formula [7] assuming that $\beta_d = 0$. Although one interpretation of this assumption is that “all of the firm’s risk is borne by the stockholders (i.e., the beta of the debt is zero)”⁸, we think that it is difficult to justify that the return on the debt is uncorrelated with the return on assets of the firm. We rather interpret formula [8] as an attempt to introduce leverage cost in the valuation: for a given risk of the assets (β_u), by using formula [8] we obtain a higher β_L (and consequently a higher K_e and a lower equity value) than with formula [7].

We label the third theory the Practitioners’ theory. The formula that relates the levered beta with the asset beta

$$[9] \quad \beta_L = \beta_u + \frac{D}{E} \beta_u \quad \text{implies that} \quad VTS = PV[K_u; T D K_u - D(K_d - R_F)]$$

It is important to notice that formula [9] is exactly formula [8] eliminating the $(1-T)$ term. We interpret formula [9] as an attempt to introduce still higher leverage cost in the valuation: for a given risk of the assets (β_u), by using formula [9] we obtain a higher β_L (and consequently a higher K_e and a lower equity value) than with formula [8].

Harris and Pringle (1985) and Ruback (1995) propose that the value creation of the tax shield is the present value of the interest tax shield discounted at the unlevered cost of equity (K_u). One straight interpretation of this assumption is that “the interest tax shields have the same systematic risk as the firm’s underlying cash flows”⁹. But another interpretation comes from analyzing the formula that relates the levered beta to the asset beta:

$$[10] \quad \beta_L = \beta_u + \frac{D}{E} (\beta_u - \beta_d)$$

It is important to notice that formula [10] is exactly formula [7] eliminating the $(1-T)$ term. We interpret formula [10] as an attempt to introduce still higher leverage cost in the valuation: for a given risk of the assets (β_u), by using formula [10] we obtain a higher β_L (and consequently a higher K_e and a lower equity value) than with formula [7].

According to Myers (1974), the value creation of the tax shield is the present value of the interest tax shield discounted at the cost of debt (K_d). The argument is that the risk of the tax saving arising from the use of debt is the same as the risk of the debt.

The sixth theory is that of Miles and Ezzell (1980). Although Miles and Ezzell do not mention what should be the discounted value of the tax shield, their formula relating the required return to equity with the required return for the unlevered company [$K_e = K_u + (D/E) (K_u - K_d) [1 + K_d (1-T)] / (1+K_d)$] implies that $VTS = PV[K_u; T D K_d] (1+K_u)/(1+K_d)$. For a firm with a fixed debt target [$D/(D+E)$] they claim that the correct rate for discounting the tax saving due to debt ($K_d T D_{t-1}$) is K_d for the tax saving during the first year, and K_u for the tax savings in the following years.

The seventh theory is Miller (1977). The value of the firm is independent of its capital structure, that is, $VTS = 0$.

The eighth theory is Fernández (2001). It quantifies the leverage cost (assuming that Fernández (2004) provides the VTS without leverage costs) as $PV[K_u; D (K_d - R_F)]$. One way of interpreting this assumption is that the reduction in the value of the firm due to

⁸ See page 31 of Damodaran (1994)

⁹ Kaplan and Ruback (1995).

leverage is proportional to the amount of debt and to the difference between the required return on debt and the risk-free rate. The cost of leverage does not depend on the tax rate.¹⁰

2.2. Different expressions of WACC and $WACC_{BT}$

The corresponding expressions of WACC with their values of K_u are:

Fernández (2004):	$WACC = K_u [1 - T D / (E+D)]$
Damodaran (1994):	$WACC = K_u - D [TK_u - (1-T) (K_d - R_F)] / (E+D)$
The practitioners' method:	$WACC = K_u - D [R_F - K_d (1-T)] / (E+D)$
Harris & Pringle (1985), Ruback (1995):	$WACC = K_u - D K_d T / (E+D)$
Myers (1974):	$WACC = K_u - [VTS (K_u - K_d) + D K_d T] / (E+D)$
Miles & Ezzell (1980):	$WACC = K_u - [D K_d T (1+K_u) / (1+K_d)] / (E+D)$
Miller (1977):	$WACC = K_u$
Fernández (2001):	$WACC = K_u - D [K_u T + R_F - K_d(1 - T)] / (E+D)$

The corresponding expressions of $WACC_{BT}$ (weighted average cost of capital before tax) with the values of K_u from the previous section are:

Fernández (2004):	$WACC_{BT} = K_u - D T (K_u - K_d) / (E+D)$
Damodaran (1994):	$WACC_{BT} = K_u + D [(K_d - R_F) - T(K_u - R_F)] / (E+D)$
The practitioners' method:	$WACC_{BT} = K_u + D (K_d - R_F) / (E+D)$
Harris & Pringle (1985), Ruback (1995):	$WACC_{BT} = K_u$
Myers (1974):	$WACC_{BT} = K_u - VTS (K_u - K_d) / (E+D)$
Miles & Ezzell (1980):	$WACC_{BT} = K_u - D T K_d (K_u - K_d) / [(E+D) (1+K_d)]$
Miller (1977):	$WACC_{BT} = K_u + D T K_d / (E+D)$
Fernández (2001):	$WACC_{BT} = K_u - D [K_u T + R_F - K_d] / (E+D)$

2.3. Different expressions of the levered beta

The different expressions of β_L (levered beta) according to the various papers are:

Fernández (2004) ¹¹ :	$\beta_L = \beta_u + D (1-T) (\beta_u - \beta_d) / E$
Damodaran (1994):	$\beta_L = \beta_u + D (1-T) \beta_u / E$
The practitioners' method:	$\beta_L = \beta_u + D \beta_u / E$
Harris & Pringle (1985) ¹² , Ruback (1995):	$\beta_L = \beta_u + D (\beta_u - \beta_d) / E$
Myers (1974) ¹³ :	$\beta_L = \beta_u + (D - VTS) (\beta_u - \beta_d) / E$

In the case of a perpetuity growing at a rate g : $\beta_L = \beta_u + D [K_d (1-T) - g] (\beta_u - \beta_d) / [E (K_d - g)]$

Miles & Ezzell (1980) ¹⁴ :	$\beta_L = \beta_u + D (\beta_u - \beta_d) [1 - T K \delta / (1+K \delta)] / E$
Miller (1977):	$\beta_L = \beta_u (D+E) / E - D [\beta_d(1-T) - T R_F / P_M]$
Fernández (2001):	$\beta_L = \beta_u + D [\beta_u (1-T) - \beta_d] / E$

¹⁰ This formula can be completed with another parameter ϕ that takes into account that the cost of leverage is not strictly proportional to debt. ϕ should be lower for small leverage and higher for high leverage. Introducing this parameter, the cost of leverage is $PV[K_u; \phi D (K_d - R_F)]$.

¹¹ This formula is the same as Taggart's (2A.6) (1991), because he assumes that $\beta_d = 0$.

¹² This formula is the same as Taggart's (2C.6) (1991), because he assumes that $\beta_d = 0$.

¹³ Note that $D - VTS = V_U - E$. Copeland, T. E., T. Koller & J. Murrin (2000) say in Exhibit A.3 of their book *Valuation: Measuring and Managing the Value of Companies*, that it is not possible to find a formula that relates the levered beta to the unlevered beta. This is not true: the relationship is the one given.

¹⁴ This formula is the same as Taggart's (2B.6) (1991), because he assumes that $\beta_d = 0$.

3. The basic problem: the value of the tax shield due to the payment of interest (VTS)

Fernández (2005) shows that in a world without leverage cost, the discounted value of the tax shields for a perpetuity is DT . It is assumed that the debt's market value (D) is equal to its book value¹⁵ (N).

Table 2 reports the implications that each of the 7 theories has for the case of perpetuities. Column [1] contains the general formula for calculating the VTS according to the 8 theories. Column [2] contains the formula for calculating the VTS for perpetuities according to the 8 theories when the tax rate is positive. Column [3] contains the formula for calculating the VTS for perpetuities according to the 8 theories when there are no taxes.

It may be seen that only 2 theories accomplish formula [15], which implies $VTS = DT$. The 2 theories are: Fernández (2004) and Myers.

The other 6 theories provide a VTS lower than DT . The difference could be attributed to the leverage cost. These 6 theories could be applicable in a "real world", where leverage cost does exist. But if that is the case, leverage cost also exists when there are no taxes. In this situation (column [3] of Table 2), these theories should provide a negative VTS. That only happens in 3 of the 6 theories: Damodaran, Practitioners and Fernández (2001).

With these two conditions, we are able to eliminate 3 theories that not only do not provide us with a value of the tax shield of DT (as the candidates for a world without cost of leverage should), nor do they provide us with a negative VTS when there are no taxes (as the candidates for a world with leverage cost should). The 3 candidates eliminated due to a lack of consistent results are the following: Harris-Pringle (1985) or Ruback (1995), Miles-Ezzell (1980), and Miller (1977).

The 8 candidate theories provide a value of $VTS = 0$ if $D = 0$.

Table 2. Perpetuity. Value of the tax shield (VTS) according to the 8 theories

Theories	VTS General formula	VTS in perpetuities	
		T>0	T = 0
	[1]	[2]	[3]
1	Fernández PV[Ku; DTKu]	DT	0
2	Damodaran PV[Ku; DTKu - D(Kd-R _f)(1-T)]	DT-[D(Kd-R _f)(1-T)]/Ku < DT	- D(Kd-R _f)/Ku < 0
3	Practitioners PV[Ku; T D Kd - D(Kd- R _f)]	D[R _f -Kd(1-T)]/Ku < DT	- D(Kd-R _f)/Ku < 0
4	Harris-Pringle PV[Ku; T D Kd]	T D Kd/Ku < DT	0
5	Myers PV[Kd; T D Kd]	DT	0
6	Miles-Ezzell PV[Ku; T D Kd]/(1+Ku)/(1+Kd ₀)	TDKd(1+Ku)/[(1+Kd ₀)Ku] < DT	0
7	Miller 0	0	0
8	Fernández PV[Ku; D(KuT+ R _f - Kd)]	D(KuT+R _f - Kd)/Ku < DT	- D(Kd-R _f)/Ku < 0

necessary conditions	with leverage cost	without leverage cost
T > 0	< DT	DT
T = 0	< 0	0
Number of theories:	3	2
	Damodaran, Practitioners and Fernández (2001)	Fernández (2004) and Myers

3 theories do not accomplish the necessary conditions to be considered:
Harris-Pringle (1985) or Ruback (1995), Miles-Ezzell (1980), and Miller (1977).

¹⁵ This means that the required return to debt (Kd) is the same as the interest rate paid by the debt (r).

3.5. Analysis of competing theories in a world without cost of leverage and with constant growth

It is clear that the required return to levered equity (K_e) should be higher than the required return to assets (K_u). Table 3 shows that only Fernández (2004) provides us always with $K_e > K_u$.

Table 3. Problems of the candidate formulas to calculate the VTS in a world without cost of leverage and with constant growth

		$K_e < K_u$
(2004)	Fernández	never
	Myers	If $g > K_d(1-T)$ ¹⁶

Another problem of Myers is that $K_e < K_u$ for high g and/or high T VTS independent of unlevered beta. On top of that, according to Myers, K_e decreases when T (tax rate) increases. According to **Fernández (2004)**, K_e increases when T increases.

When the cost of debt (r) is not equal to the required return to debt (K_d), the value of the tax shield, according to **Fernández (2004)**, should be calculated as follows¹⁷:

$$VTS = PV[K_u; DTK_u + T(Nr - DK_d)]$$

We would point out again that this expression is not the PV of a cash flow, but the difference between the present values of two cash flows, each with different risk: the taxes of the company without debt and the taxes of the company with debt.

4. Differences in the valuation according to the most significant papers

4.1. Growing perpetuity with a preset debt ratio of 30%

Upon applying the above formulas to a company with $FCF_1 = 100$, $K_u = 10\%$, $K_d = 7\%$, $[D/(D+E)] = 30\%$, $T = 35\%$, $R_F = 5\%$, and $g = 5\%$, we obtain the values given in Table 4. The value of the unlevered company (V_u) is 2,000 in all cases. Note that, according to Myers, $K_e < K_u = 10\%$, which makes no sense. Neither does it make any sense that $VTS > D$, which is what happens when $g > K_d(1-T)$; in the example, when $g > 4.55\%$.

¹⁶ $K_e < K_u$ if $VTS > D$. $DTK_d / (K_d - g) > D$ implies $g > K_d(1-T)$. It can also be expressed as $V_u < E$.

¹⁷ See Fernández (2001).

Table 4. Example of a company valuation
 $FCF_1 = 100$, $K_u = 10\%$, $K_d = 7\%$, $[D/(D+E)] = 30\%$, $T = 35\%$, $R_F = 5\%$, and $g = 5\%$

	Fernández	Myers	Miller	Miles-Ezzell	Harris-Pringle	Damodaran	Practitioners
WACC	8.950%	8.163%	10.000%	9.244%	9.265%	9.340%	9.865%
K_e	10.836%	9.711%	12.336%	11.256%	11.286%	11.393%	12.143%
$WACC_{BT}$	9.685%	8.898%	10.735%	9.979%	10.000%	10.075%	10.600%
E+D	2,531.65	3,162.06	2,000.00	2,356.05	2,344.67	2,304.15	2,055.50
V_u	2,000.00	2,000.00	2,000.00	2,000.00	2,000.00	2,000.00	2,000.00
E	1,772.15	2,213.44	1,400.00	1,649.23	1,641.27	1,612.90	1,438.85
D	759.49	948.62	600.00	706.81	703.40	691.24	616.65
VTS	531.65	1,162.06	0.00	356.05	344.67	304.15	55.50
ECF	103.42	104.27	102.70	103.18	103.17	103.11	102.77

If we make changes to the growth rate, Tables 5 to 7 show the valuation's basic parameters at different values of the growth rate g .

Table 5 shows that the company's WACC is independent of growth, according to all the theories except Myers'. According to Myers, the WACC falls when growth increases and is equal to growth when $g = K_d [D(1-T)+E]/(E+D)$; in the example, when $g = 6.265\%$.

Table 5. WACC at different growth rates g . $[D/(D+E)] = 30\%$

g	Fernández	Myers	Miller	Miles-Ezzell	Harris-Pringle	Damodaran	Practitioners
0%	8.95%	8.95%	10.00%	9.24%	9.27%	9.34%	9.87%
2%	8.95%	8.82%	10.00%	9.24%	9.27%	9.34%	9.87%
4%	8.95%	8.53%	10.00%	9.24%	9.27%	9.34%	9.87%
6%	8.95%	7.06%	10.00%	9.24%	9.27%	9.34%	9.87%

Table 6 show that the VTS according to the Fernández (2004) and according to Myers are equal for a perpetuity (when there is no growth). When there is growth, the value of the VTS according to Myers is higher than the VTS according to Fernández (2004). All the other theories give values lower than the Fernández (2004) theory. According to Myers, the company's value is infinite for growth rates equal or greater than $g = K_d [D(1-T)+E]/(E+D)$; in the example, when $g \geq 6.265\%$.

Table 6. VTS at different growth rates g . $[D/(D+E)] = 30\%$

g	Fernández	Myers	Miller	Miles-Ezzell	Harris-Pringle	Damodaran	Practitioners
0%	117.3	117.3	0.0	81.7	79.3	70.7	13.7
2%	188.8	215.4	0.0	130.4	126.5	112.4	21.5
4%	353.5	540.8	0.0	240.1	232.7	206.0	38.4
6%	889.8	6,934.0	0.0	582.2	562.8	494.0	87.3
7%	1,794.9	∞	0.0	1,122.2	1,081.7	940.2	157.1

Table 7 shows that the required return to equity is independent of growth according to all the theories except Myers'. According to Myers, K_e falls when growth increases and is equal to K_u when $g = K_d(1-T)$. In the example, when $g = 4.55\%$. Obviously, this makes no sense.

Table 7. Ke at different growth rates g. $[D/(D+E)] = 30\%$

g	Fernández	Myers	Miller	Miles-Ezzell	Harris-Pringle	Damodaran	Practitioners
0%	10.8%	10.8%	12.3%	11.3%	11.3%	11.4%	12.1%
2%	10.8%	10.7%	12.3%	11.3%	11.3%	11.4%	12.1%
4%	10.8%	10.2%	12.3%	11.3%	11.3%	11.4%	12.1%
5%	10.8%	9.7%	12.3%	11.3%	11.3%	11.4%	12.1%
6%	10.8%	8.1%	12.3%	11.3%	11.3%	11.4%	12.1%

If the debt ratio is changed, Tables 8 and 9 show the valuation's basic parameters at different debt ratios.

Table 8 shows the VTS at different debt ratios according to the various theories. The value of the VTS according to Myers is higher than the VTS according to Fernández (2004). All the other theories give values lower than the Fernández (2004) theory. It can be seen that the VTS according to Myers becomes infinite for a debt ratio $D/(D+E) = (Kd-g) / (T Kd)$, in our example, 81.63%.

Table 8. Present value of the tax shield (VTS) at different debt ratios (g=5%). $[D/(D+E)] = 30\%$

D/(D+E)	Fernández	Myers	Miller	Miles-Ezzell	Harris-Pringle	Damodaran	Practitioners
0%	0.0	0.0	0.0	0.0	0.0	0.0	0.0
20%	325.6	649.0	0.0	224.1	217.3	193.0	36.7
40%	777.8	1,921.6	0.0	504.7	487.6	427.2	74.7
60%	1,448.3	5,547.2	0.0	866.3	832.9	717.4	114.2
70%	1,921.6	12,035.1	0.0	1,089.4	1,044.1	890.2	134.5
80%	2,545.5	98,000.0	0.0	1,350.0	1,289.5	1,086.4	155.2
90%	3,405.4	∞	0.0	1,658.7	1,577.8	1,311.3	176.3
100%	4,666.7	∞	0.0	2,030.1	1,921.6	1,571.4	197.8

Table 9. Required return to equity (Ke) at different debt ratios (g=5%)

D/(D+E)	Fernández	Myers	Miller	Miles-Ezzell	Harris-Pringle	Damodaran	Practitioners
0%	10.00%	10.00%	10.00%	10.00%	10.00%	10.00%	10.00%
20%	10.49%	9.83%	11.36%	10.73%	10.75%	10.81%	11.25%
40%	11.30%	9.55%	13.63%	11.95%	12.00%	12.17%	13.33%
60%	12.93%	8.99%	18.18%	14.40%	14.50%	14.88%	17.50%
80%	17.80%	7.30%	31.80%	21.73%	22.00%	23.00%	30.00%
90%	27.55%	3.93%	59.05%	36.38%	37.00%	39.25%	55.00%
95%	47.05%	-2.82%	113.55%	65.69%	67.00%	71.75%	105.00%

4.2. Growing perpetuity with preset debt

The hypotheses of Table 10 are identical to those of Table 4, with the sole difference that the initial debt level is set at 759.49 (instead of the debt ratio of 30%). The value of the unlevered company (V_u) is 2,000 in all cases. Note that, according to Myers, $K_e < K_u = 10\%$, which does not make much sense.

Table 10. Example of a company valuation
 $FCF_1 = 100$, $K_u = 10\%$, $K_d = 7\%$, $D = 759.49$, $T = 35\%$, $R_F = 5\%$, and $g = 5\%$

	Fernández	Myers	Miller	Miles-Ezzell	Harris-Pringle	Damodaran	Practitioners
WACC	8.950%	8.413%	10.000%	9.197%	9.216%	9.284%	9.835%
K_e	10.836%	9.764%	13.337%	11.372%	11.413%	11.568%	12.901%
$WACC_{BT}$	9.685%	9.048%	10.930%	9.978%	10.000%	10.081%	10.734%
E+D	2,531.65	2,930.38	2,000.00	2,382.59	2,372.15	2,334.18	2,068.35
V_u	2,000.00	2,000.00	2,000.00	2,000.00	2,000.00	2,000.00	2,000.00
E	1,772.15	2,170.89	1,240.51	1,623.09	1,612.66	1,574.68	1,308.86
D	759.49	759.49	759.49	759.49	759.49	759.49	759.49
VTS	531.65	930.38	0.00	382.59	372.15	334.18	68.35
ECF	103.42	103.42	103.42	103.42	103.42	103.42	103.42
D/(D+E)	30.00%	25.92%	37.97%	31.88%	32.02%	32.54%	36.72%

Appendix 1

In a world with no leverage cost the value of tax shields is $PV[Ku; D T Ku]$

Fernández (2005) shows that the VTS in a world with no leverage cost is the tax rate times the debt, plus the tax rate times the present value of the net increases of debt. This expression is the difference between the present values of two different cash flows, each with its own risk: the present value of taxes for the unlevered company and the present value of taxes for the levered company. The critical parameter for calculating the value of tax shields is the present value of the net increases of debt. It may vary for different companies, but in some special circumstances it may be calculated.

For perpetual debt, the value of tax shields is equal to the tax rate times the value of debt. When the company is expected to repay the current debt without issuing new debt, Myers (1974) applies, and the value of tax shields is the present value of the interest times the tax rate, discounted at the required return to debt. If the correct discount rate for the increases of debt is the required return to the unlevered company, then formula (28) of Fernández (2004) applies.

One could say “in practice I do not see why the approach of working out the present value of tax shields themselves would necessarily be wrong, provided the appropriate discount rate was used (reflecting the riskiness of the tax shields)”. The problem here is that it is hard to evaluate the riskiness of tax shields because it is the difference of two present flows (the taxes paid by the unlevered company and those paid by the levered company), each with different risk.

To evaluate the riskiness of tax shields is as hard as to evaluate the riskiness of the difference between the expected equity cash flow of Microsoft and the expected equity cash flow of GE. We may evaluate the riskiness of the expected equity cash flows of each company, but it is difficult (and I suggest that it does not make any sense) to try to evaluate the riskiness of the difference of the two expected equity cash flows.

I provide a more general expression for the value of tax shields than the one given in Fernández (2004). The title of Fernández (2004) still applies: The value of tax shields is not equal to the present value of tax shields, but the difference between the present values of two different cash flows, each with its own risk: the present value of taxes for the unlevered company and the present value of taxes for the levered company. This correction shows that one of the conclusions of Fernández (2004) is valid only for specific situations. More specifically, formula (28) $VTS = PV[Ku; D \cdot T \cdot Ku]$ is valid only under the assumption that the debt increases are as risky as the free cash flows.

The key equations of Fernández (2004) that are affected by using the expected value notation are¹⁸:

$$ECF_t = PAT_{L_t} - \Delta NFA_t - \Delta WCR_t + \Delta D_t \quad (5a)$$

Notation being, ECF = Equity Cash Flow; FCF = Free Cash Flow; PAT = Profit after Tax = $\Delta WCR_t = WCR_t - WCR_{t-1}$ = Increase of Working Capital Requirements in period t ; $\Delta NFA_t = NFA_t - NFA_{t-1}$ = Increase of Net Fixed Assets in period t ; $\Delta D_t = D_t - D_{t-1}$ = Increase of Debt in period t .

¹⁸ I will refer with an “a” to the equations of Fernández (2004).

Apéndice 1 (continuación)

$$FCF_t = PAT_{ut} - \Delta NFA_t - \Delta WCR_t \quad (7a)$$

$$Taxes_{U_t} = [T/(1+T)] PAT_u = [T/(1+T)] (FCF_t + \Delta NFA_t + \Delta WCR_t) \quad (9a)$$

$$Taxes_{L_t} = [T/(1+T)] (ECF_t + \Delta NFA_t + \Delta WCR_t - \Delta D_t) \quad (12a)$$

$Taxes_U$ and $Taxes_L$ are the taxes paid by the unlevered company and those paid by the levered company.

$PV_0[\cdot]$ is the present value operator. The present values at $t=0$ of equations (9a) and (12a) are:

$$G_{U_0} = [T/(1+T)] (V_{u_0} + PV_0[\Delta NFA_t + \Delta WCR_t]) \quad (11a)$$

$$G_{L_0} = [T/(1+T)] (E_0 + PV_0[\Delta NFA_t + \Delta WCR_t] - PV_0[\Delta D_t]) \quad (14a)$$

G_U is the present value of the taxes paid by the unlevered company and G_L is the present value of the taxes paid by the levered company.

The value of the tax shield (VTS) comes from the difference between (11a) and (14a):

$$VTS_0 = G_{U_0} - G_{L_0} = [T/(1+T)] (V_{u_0} - E_0 + PV_0[\Delta D_t]) \quad (15a)$$

As, according to equation (1), $V_{u_0} - E_0 = D_0 - VTS_0$, then

$VTS_0 = [T/(1+T)] (D_0 - VTS_0 + PV_0[\Delta D_t])$. And the value of tax shields is:

$$VTS_0 = T \cdot D_0 + T \cdot PV_0[\Delta D_t] \quad (16a)$$

Equation (16a) is valid for perpetuities and for companies with any pattern of growth. More importantly, this equation shows that the value of tax shields depends only upon the nature of the stochastic process of the net increase of debt. The problem of equation (16a) is how to calculate $PV_0[\Delta D_t]$, which requires knowing the appropriate discount rate to apply to the expected increase of debt.¹⁹

We may not know what are the correct values of G_U and G_L , but we do know the value of the difference, provided we can value $PV_0[\Delta D_t]$, the present value of the net debt increases.

1. VTS in specific situations

To develop a better understanding of the result in (16a), we apply it in specific situations and show how this formula is consistent with previous formulae under restrictive scenarios.

¹⁹ If the nominal value of debt (N) is not equal to the value of debt (D), because the interest rate (r) is different from the required return to debt flows (K_d), equation (16a) is: $VTS_0 = T \cdot D_0 + T \cdot PV_0[\Delta N_t]$. The relationship between D and N is: $D_0 = PV_0[\Delta N_t] + PV_0[N_t \cdot r_t]$.

1.1. Perpetual debt

If the debt is a constant perpetuity (a consol), $PV_0[\Delta D_t] = 0$, and $VTS_0 = T \cdot D_0$

1.2. Debt of one-year maturity but perpetually rolled over

As in the previous case, $E\{D_t\} = D_0$, but the debt is expected to be rolled over every year. The appropriate discount rate for the cash flows due to the existing debt is Kd^{20} . Define K_{ND} as the appropriate discount rate for the new debt (the whole amount) that must be obtained every year, then:

Present value of obtaining the new debt every year²¹ $= D_0 / K_{ND}$

Present value of the principal repayments at the end of every year²² $= D_0 (1 + K_{ND}) / [(1 + Kd) K_{ND}]$

$PV_0[\Delta D_t]$ is the difference between the last two expressions. Therefore:

$$PV_0[\Delta D_t] = -D_0 (K_{ND} - Kd) / [(1 + Kd) K_{ND}] \quad (14)$$

In a constant perpetuity ($E\{FCF_t\} = FCF_0$), it may be reasonable that, if we do not expect credit rationing, $K_{ND} = Kd$, which means that the risk associated with the repayment of the current debt and interest (Kd) is equivalent to the risk associated with obtaining an equivalent amount of debt at the same time (K_{ND}).

1.3. Debt is proportional to the Equity value

This is the assumption made by Miles and Ezzell (1980) and Arzac and Glosten (2005), who show that if $D_t = L \cdot E_t$, then the value of tax shields for perpetuities growing at a constant rate g is:

$$VTS_0 = \frac{D_0 Kd T (1 + Ku)}{(Ku - g) (1 + Kd)} \quad (50)$$

Substituting (50) in (16a), we get:

$$PV_0[\Delta D_t] = D_0 \frac{(Kd - Ku) + g(1 + Kd)}{(Ku - g)(1 + Kd)} \quad (51)$$

For the no growth case ($g = 0$), equation (51) is:

$$PV_0[\Delta D_t] = D (Kd - Ku) / [Ku(1 + Kd)] < 0.$$

²⁰ We use Kd so as not to complicate the notation. It should be Kd_t , a different rate following the yield curve. Using Kd we may also think of a flat yield curve.

²¹ Present value of obtaining the new debt every year $= D / (1 + K_{ND}) + D / (1 + K_{ND})^2 + D / (1 + K_{ND})^3 + \dots$ because $D = E\{D_t\}$, where D_t is the new debt obtained at the end of year t (beginning of $t+1$).

²² The present value of the principal repayment at the end of year 1 is $D / (1 + Kd)$
The present value of the principal repayment at the end of year 2 is $D / [(1 + Kd)(1 + K_{ND})]$
The present value of the principal repayment at the end of year t is $D / [(1 + Kd)(1 + K_{ND})^{t-1}]$
Because $D = E\{D_t\}$, where D_t is the debt repayment at the end of year t .

Comparing this expression with equation (14), it is clear that Miles and Ezzell imply that $K_{ND} = K_u$. However, to assume $D_t = L \cdot E_t$ is not a good description of the debt policy of any company because:

1. If the company pays a dividend Div_t , simultaneously the company should reduce debt in an amount $\Delta D_t = -L \cdot Div_t$
2. If the equity value increases, then the company should increase its debt, while if the equity value decreases, then the company should reduce its debt. If the equity value is such that $L \cdot E_t > (\text{Assets of the company} - \text{Book Value of equity})$, then the company should hold excess cash only for the sake of complying with the debt policy.

1.4. Debt increases are as risky as the free cash flows

In this situation, the correct discount rate for the expected increases of debt is K_u , the required return to the unlevered company. In the case of a constant growing perpetuity, $PV_0[\Delta D_t] = g \cdot D_0 / (K_u - g)$, and the VTS is equation (28) in Fernández (2004):

$$VTS_0 = T \cdot K_u \cdot D_0 / (K_u - g) \quad (28)$$

1.5. The company is expected to repay the current debt without issuing new debt

In this situation, the appropriate discount rate for the negative ΔD_t (because they are principal payments) is K_d , the required return to the debt. In this situation, Myers (1974) applies: $PV_0[\Delta D_t] = PV_0[E\{\Delta D_t\}; K_d]$, and the VTS is:

$$VTS_0 = D_0 \cdot T + T \cdot PV_0[E\{\Delta D_t\}; K_d] \quad (18)$$

For a company that is expected to repay the current debt without issuing new debt, the value of the debt today is: $D_0 = PV_0[E\{D_{t-1}\} \cdot K_d - E\{\Delta D_t\}; K_d]$.

Substituting this expression in (18), we get the Myers (1974) formula:

$$VTS_0 = PV_0[T \cdot E\{D_{t-1}\} \cdot K_d; K_d]$$

2. Value of net debt increases implied by the alternative theories

Table 1 summarizes the implications of several approaches for the value of tax shields. From equation (16a) the present value of the increases of debt is:

$$PV_0[\Delta D_t] = (VTS_0 - T \cdot D_0) / T$$

Applying this equation to the theories mentioned, we may construct the predictions that each of these theories have for $PV_0[\Delta D_t]$.

As we have already argued, Myers (1974) should be used when the company will not issue new debt; and Fernández (2004) when the company expects to issue new debt in the future and we expect the increases of debt be as risky as the free cash flow. Modigliani-

Miller may be applied only if the debt is risk-free. Miles-Ezzell (1980) may be used only if debt will be always a multiple of the equity value $D_t = L \cdot E_t$,

Table 1. Comparison of value of tax shields (VTS) in perpetuities

Only three out of the seven approaches correctly compute the value of the tax shield in perpetuities as DT.

The other four theories imply a lower value of the tax shield than DT.

Theories	VTS	PV ₀ [ΔD _t] for constant growing perpetuities at a rate g
Correct method	$D \cdot T + T \cdot PV_0[\Delta D_t]$	
Damodaran (1994)	$PV[E\{D \cdot T \cdot Ku - D(Kd - R_F)(1-T)\}; Ku]$	$\frac{g \cdot D_0}{Ku - g} - \frac{D_0(Kd - R_F)(1-T)}{Ku - g} - \frac{T}{T}$
Practitioners	$PV[E\{D \cdot T \cdot Kd - D(Kd - R_F)\}; Ku]$	$\frac{g \cdot D_0}{Ku - g} - \frac{D_0(Ku - Kd)}{Ku - g} - \frac{D_0(Kd - R_F)}{(Ku - g)T}$
Harris-Pringle (1985), Ruback (1995)	$PV[E\{D \cdot T \cdot Kd\}; Ku]$	$\frac{g \cdot D_0}{Ku - g} - \frac{D_0(Ku - Kd)}{Ku - g}$
Myers (1974)	$PV[E\{D \cdot T \cdot Kd\}; Kd]$	$\frac{g \cdot D_0}{Kd - g}$
Miles-Ezzell (1980)	$PV[E\{D \cdot T \cdot Kd\}; Ku] (1+Ku) / (1+Kd)$	$\frac{g \cdot D_0}{Ku - g} - \frac{D_0(Ku - Kd)}{(Ku - g)(1 + Kd)}$
Modigliani-Miller (1963)	$PV[E\{D \cdot T \cdot R_F\}; R_F]$	$\frac{g \cdot D_0}{R_F - g}$
Fernández (2004)	$PV[E\{D \cdot T \cdot Ku\}; Ku]$	$\frac{g \cdot D_0}{Ku - g}$

Ku = unlevered cost of equity; Kd = required return to debt; T = corporate tax rate; D = debt value; R_F = risk-free rate; PV[E{D·T·Ku}; Ku] = present value of the expected value of D·T·Ku discounted at the rate Ku

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