IESE Business School
University of Navarra

CIIF

Working Paper

WP No 604

July, 2005

DISCOUNTED CASH FLOW VALUATION METHODS: EXAMPLES OF PERPETUITIES, CONSTANT GROWTH AND GENERAL CASE

Pablo Fernández*

* Professor of Financial Management, PricewaterhouseCoopers Chair of Finance, IESE

IESE Business School – University of Navarra
Av. Pearson, 21 – 08034 Barcelona, Spain. Tel.: (+34) 93 253 42 00 Fax: (+34) 93 253 43 43
Camino del Cerro del Águila, 3 (Ctra. de Castilla, km 5,180) – 28023 Madrid, Spain. Tel.: (+34) 91 357 08 09 Fax: (+34) 91 357 29 13

The CIIF, International Center for Financial Research, is an interdisciplinary center with an international outlook and a focus on teaching and research in finance. It was created at the beginning of 1992 to channel the financial research interests of a multidisciplinary group of professors at IESE Business School and has established itself as a nucleus of study within the School's activities.

Ten years on, our chief objectives remain the same:

- Find answers to the questions that confront the owners and managers of finance companies and the financial directors of all kinds of companies in the performance of their duties
- Develop new tools for financial management
- Study in depth the changes that occur in the market and their effects on the financial dimension of business activity

All of these activities are programmed and carried out with the support of our sponsoring companies. Apart from providing vital financial assistance, our sponsors also help to define the Center's research projects, ensuring their practical relevance.

The companies in question, to which we reiterate our thanks, are:
Aena, A.T. Kearney, Caja Madrid, Fundación Ramón Areces, Grupo Endesa, Telefónica and Unión Fenosa.

http://www.iese.edu/ciif/

DISCOUNTED CASH FLOW VALUATION METHODS:
EXAMPLES OF PERPETUITIES, CONSTANT GROWTH AND GENERAL CASE

Abstract

This paper explores the discounted cash flow valuation methods. We start the paper with the simplest case: no-growth, perpetual-life companies. Then we will study the continuous growth case and, finally, the general case.

The different concepts of cash flow used in company valuation are defined: equity cash flow (ECF), free cash flow (FCF), and capital cash flow (CCF). Then the appropriate discount rate is determined for each cash flow, depending on the valuation method used.

Our starting point will be the principle by which the value of a company's equity is the same, whichever of the four traditional discounted cash flow formulae is used. This is logical: given the same expected cash flows, it would not be reasonable for the equity's value to depend on the valuation method.

JEL classification: G12; G31; G32

Keywords: discounted cash flow valuation, cash flow valuation, value of tax shields, required return to equity

DISCOUNTED CASH FLOW VALUATION METHODS:
EXAMPLES OF PERPETUITIES, CONSTANT GROWTH AND GENERAL CASE

1. Introduction

Initially, it is assumed that the debt's market value (D) is equal to its book value[1] (N). Section 5 discusses the case in which the debt's book value (N) is not equal to its market value (D), as is often the case, and section 6 analyzes the impact of the use of simplified formulae to calculate the levered beta.

Section 7 addresses the valuation of companies with constant growth, and section 11 discusses the general case in company valuation.

2. Company valuation formulae. Perpetuities

The cash flows generated by the company are perpetual and constant (there is no growth). The company must invest in order to maintain its assets at a level that enables it to ensure constant cash flows: this implies that the book depreciation is equal to the replacement investment.

We start with a numerical example, to help the reader become familiar with the concepts.

Income statements and cash flows:	
Margin	800
Interest paid (I)	225
Profit before tax (PBT)	575
Taxes (T = 40%)	230
Profit after tax (PAT)	345
+ Depreciation	200
- Investment in fixed assets	-200
ECF	345

[1] This means that the required return to debt (Kd) is equal to the interest rate paid by the debt (r).

FCF = ECF + I (1-T) = 345 + 225 (1 - 0.40) = 480
CCF = ECF + I = 345 + 225 = 570
R_F =12%. P_M (Market risk premium) = 8%. Assets' beta (βu) = 1. Equity's beta (β_L) = 1,375. Cost of debt = 15%.
Debt's beta = 0.375. Equity market value (E) = 1,500. Equity book value = 800. Debt (D) = 1,500

2.1. Calculating the company's value from the equity cash flow (ECF)

The following pages explain the four discounted cash flow methods most commonly used for company valuation in the case of perpetuities. Formula [1] indicates that the equity's value (E) is the present value of the expected equity cash flow (ECF) discounted at the required return to equity (Ke). The required return to equity (Ke) is often called "cost of equity".

Formula [1] is equivalent to the equation we would use to calculate the value of a perpetual bond. This type of bond gives its holder constant cash flows that remain perpetually the same. In order to calculate the value of this bond, we would discount the payment of the regular coupon at the market interest rate for this type of debt. Likewise, the value of a company's equity (E) is the present value of the cash flows that would be paid to its owners (ECF), discounted at that company's required return to equity[2] (Ke).

[1] E = ECF / Ke

In the example: E = 345/23% = 1,500 because $Ke = R_F + \beta_L P_M$ = 12% + 1,375 x 8% = 23%

Consequently, the company's value[3] will be equal to the value of the equity (E) plus the value of the debt (D):

[2] E + D = ECF / Ke + I / Kd where D = I / Kd

In the example: E + D = 345/0.23 + 225/0.15 = 1,500 + 1,500 = 3,000

The market value of the debt (D) is equal to its book value[4]. The interest paid (I) is equal to the book value of the debt (D) times the cost of debt (Kd). The beta of the debt is calculated following the CAPM:

$Kd = R_F + \beta_d P_M$; 15% = 12% + 0.375 x 8%

2.2. Calculating the company's value from the free cash flows (FCF)

Formula [3] proposes that the value of the debt today (D) plus that of the equity (E) is the present value of the expected free cash flows (FCF) that the company will generate, discounted at the weighted cost of debt and equity after tax (WACC).

[3] E + D = FCF / WACC

---

[2] It is important to remember that the required return (or cost of capital) depends on the funds' use and not on their source.

[3] The value of the equity (E) plus the value of the debt (D) is usually called company's value or value of the company.

[4] For the moment, we will assume that the cost of debt (the interest rate paid by the company) is identical to the required return to debt.

The expression that relates the FCF with the ECF is:

[4] ECF = FCF - D Kd (1-T)

In the example: ECF = FCF - D Kd (1-T) = 480 - 1,500 x 0.15 x (1- 0.4) = 345

As [2] and [3] must be the same, substituting [4] gives:

(E+D) WACC = E Ke + D Kd (1-T)

Consequently, the definition of WACC, or “weighted average cost of capital”, is:

[5] $$WACC = \frac{E\ Ke + D\ Kd\ (1 - T)}{E + D}$$

Note that the WACC is the discount rate that ensures that the value of the company (E+D) obtained using [3] is the same as that obtained using [2].

In the example: E+D = 480/0.16 = 3,000; WACC = [1,500 x 0.23 + 1,500 x 0.15 x (1 - 0.4)] / (1,500 + 1,500) =16%

2.3. Calculating the company’s value from the capital cash flows (CCF)

Formula [6] uses the capital cash flows as their starting point and proposes that the value of the debt today (D) plus that of the equity (E) is equal to the capital cash flow (CCF) discounted at the weighted cost of debt and equity before tax[5] ($WACC_{BT}$). The CCF is the cash flow available for all holders of the company’s instruments, whether these are debt or capital, and is equal to the equity cash flow (ECF) plus the debt cash flow (CFd), which, in the case of perpetuities, is the interest paid on the debt (I).

[6] E + D = CCF / $WACC_{BT}$

The expression that relates the CCF with the ECF and the FCF is:

[7] CCF =ECF + CFd = ECF + D Kd = FCF + D Kd T

In the example: CCF = ECF + CFd = 345 + 225 = 570; CCF = FCF + IT = 480 + 225 x 0.4 = 570

As [2] must be equal to [6], using [7] gives: (E+D) $WACC_{BT}$ = E Ke +D Kd

And, consequently, the definition of $WACC_{BT}$ is:

[8] $$WACC_{BT} = \frac{E\ Ke + D\ Kd}{E + D}$$

Note that the expression of $WACC_{BT}$ is obtained by making [2] equal to [6]. $WACC_{BT}$ is the discount rate that ensures that the value of the company obtained using the two expressions is the same.

---

[5] BT means “before tax“.

In the example: E + D = 570/0.19 = 3,000. Because CCF = 345 + 225 = 570
and $WACC_{BT}$ = (1,500 x 0.23 + 1,500 x 0.15) / (1,500 + 1,500) = 19%

2.4. Adjusted present value (APV)

The formula for the adjusted present value [9] indicates that the value of the debt today (D) plus that of the equity (E) of the levered company is equal to the value of the equity of the unlevered company Vu (FCF/Ku) plus the value of the tax shields due to interest payments:

[9] E + D = Vu + value of the tax shields = FCF / Ku + value of the tax shields

In the case of perpetuities:

[10] VTS = Value of the tax shields = DT

In the example: E + D = 480/0.2 + 1,500 x 0.4 = 3,000

Expression [10] is demonstrated in section 3. This entails not considering leverage costs and is discussed further on in Fernández (2004 and 2005).

By equaling formulae [2] and [9] and taking into account [10] and [3], it is possible to obtain the relationship between Ku and WACC:

[11] WACC = Ku [E + D(1-T)] / (E+D)

In the example: WACC = 0.2 x [1,500 + 1,500 x (1 - 0.4)] / (1,500 + 1,500) = 16%

Formula [11] indicates that with tax, in a company with debt, WACC is always less than Ku, and the higher the leverage, the smaller it is. Note also that WACC is independent of Kd and Ke (it depends on Ku). This may seem unintuitive, but it is logical. Note that when D = 0, WACC = Ku. When E = 0, WACC = Ku (1 - T).

By substituting [5] in [11], we can obtain the relationship between Ku, Ke and Kd:

[12] $$Ku = \frac{E\ Ke + D\ Kd(1-T)}{E + D\,(1-T)} = \frac{E\ Ke + D\ Kd\,(1-T)}{Vu}$$

In the example: Ku = 20% = [1,500 x 0.23 + 1,500 x 0.15 x (1 - 0.4)] / [1,500 + 1,500 x (1 - 0.4)]

2.5. Use of the CAPM and expression of the levered beta

Formulae [13], [14] and [15] are simply the relationship, according to the capital asset pricing model (CAPM), between the required return to equity of the unlevered company (Ku), the required return to equity of the levered company (Ke), and the required return to debt (Kd), with their corresponding betas (β):

[13] $Ku = R_F + \beta_U P_M$

[14] $Ke = R_F + \beta_L P_M$

[15] $Kd = R_F + \beta_d P_M$

R_F = Risk-free interest rate. β_d = Beta of the debt. β_U = Beta of the equity of the unlevered company.

β_L = Beta of the equity of the levered company. P_M = Market risk premium.

In the example: Ku = 12 + 1 x 8 = 20%; Ke = 12 + 1.375 x 8 = 23%; Kd = 12 + 0.375 x 8 = 15%

Another way of expressing [12] is[6], isolating Ke:

[16] Ke = Ku + [(Ku – Kd) D (1 – T)] / E

Substituting Ke, Ku and Kd in this equation with expressions [13], [14] and [15], we obtain:

[17] $$\beta_L = \frac{\beta_U\,[E + D(1-T)] - \beta_d\,D(1-T)}{E}$$

In the example: β_L = 1.375 = (1 x [1,500+1,500 x 0.6] - 0.375 x 1,500 x 0.6) / 1,500

3. VTS in perpetuities. Tax risk in perpetuities

As we stated in the introduction, the value of the levered company (VL = E + D) obtained with all four methods is identical, as shown in diagram form in Figure 1[7]. However, it is important to remember that by forcing fulfillment of the adjusted present value formulae [9] and [10], we are accepting that the company's total value (debt, equity and tax) is independent of leverage, that is, there are no leverage-generated costs (there is no reduction in the expected FCF nor any increase in the company's risk).

In a world without leverage cost, the following relationship holds:

[18] Vu + Gu = E + D + GL

Gu is the present value of the taxes paid by the unlevered company. GL is the present value of the taxes paid by the levered company. The VTS (value of the tax shields) is:

[19] VTS = Gu - GL

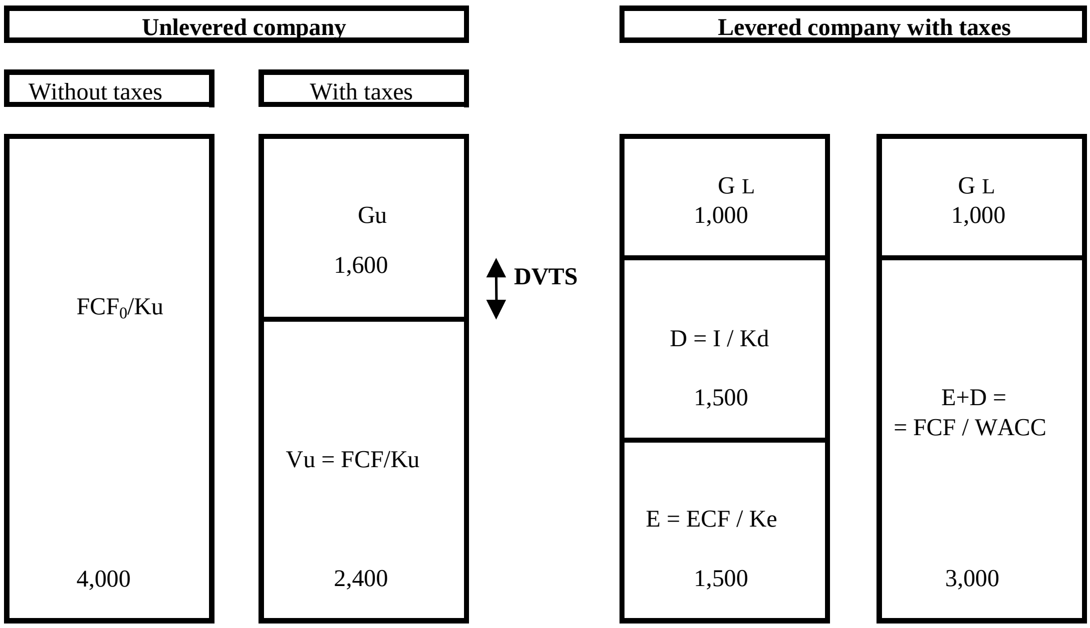
In a perpetuity, the profit after tax (PAT) is equal to equity cash flow: PAT = ECF. This is because, in a perpetuity, depreciation must be equal to reinvestment in order to keep the cash flow generation capacity constant.

We will call FCF_0 the company's free cash flow if there were no taxes, i.e.: PBTu = FCF_0, then: FCF = FCF_0 (1- T).

[6] This formula "seems" to indicate that if taxes are increased, Ke decreases. However, this is not true. Ke does not depend on T. In the formula, Ku, Kd and D do not depend on T, and neither does Ke. However, E does depend on T. Performing simple algebraic operations, it is possible to verify that if taxes increase by an amount ΔT, the decrease in the shares' value (ΔE), is: ΔE = E ΔT / (1-T).

[7] Note that we include a third beneficiary element in the company: the State, whose revenues consist of taxes.

Figure 1. Distribution of the company's total value between shareholders, bondholders and the Government. Without leverage costs



E + D = Vu + VTS

For the unlevered company (D = 0):

[20] $Taxes_U = T\ PBTu = T\ FCF_0 = T\ FCF / (1\text{-}T)$

Consequently, the taxes of the unlevered company have the same risk as FCF_0 (and FCF), and must be discounted at the rate Ku**.** The required return to tax in the unlevered company (K_{TU}) is equal to the required return to equity in the unlevered company (Ku)[8].

[21] $\mathbf{K_{TU} = Ku}$

The present value of the taxes of the unlevered company is:

[22] $Gu = T\ FCF / [(1\text{-}T)\ Ku] = T\ Vu / (1\text{-}T)$

For the levered company:

[23] $Taxes_L = T\ PBT_L = T\ PAT_L / (1\text{-}T) = T\ ECF / (1\text{-}T)$

Consequently, the taxes of the levered company have the same risk as the ECF and must be discounted at the rate Ke. Thus, in the case of perpetuities, the tax risk is identical to the equity cash flow risk and –consequently– the required return to tax in the levered company (K_{TL}) is equal to the required return to equity (Ke)[9].

[24] $\mathbf{K_{TL} = Ke}$

[8] This is only true for perpetuities.
[9] This is only true for perpetuities.

The present value of the taxes of the levered company, that is, the value of the State's interest in the company is[10]:

[25] G_L = T ECF / [(1-T) Ke] = T E / (1-T)

The increase in the company's value due to the use of debt is ***not*** the present value of the tax shields due to interest payments, but the difference between Gu and G_L, which are the present values of two cash flows with different risks:

[26] Gu - G_L = [T / (1-T)] (Vu – E)

As Vu – E = D – VTS, this gives:

[10] VTS = Value of the tax shields = DT

In the example: FCF_0 = 800; FCF = 480; PBTu = 800; $Taxes_U$ = 320; ECF = 345; $Taxes_L$ = 230. Gu = 1,600, GL = 1,000

DT = 600 = 1,600 – 1,000

Figure 1 shows how Vu + DT = D + E.

It is important to note that the value of the tax shields **(VTS)** is not (and this is the main error of many books and papers on this topic) the PV of the tax shields, but the difference between the PVs of two flows with different risk: the PV of the taxes paid in the unlevered company (Gu) and the PV of the taxes paid in the levered company (G_L). Formula [10] is the difference between the two PVs. Obviously, the flow of taxes paid in the levered company is smaller and riskier than the flow of taxes paid in the unlevered company.

4. Examples of companies without growth

Table 1 shows the valuation of six different companies without growth. The companies differ in the tax rate, the cost of debt and the size of debt. Column [A] corresponds to the company without debt and without taxes. Column [B] corresponds to the same company paying a tax rate of 35%. Column [C] corresponds to a company with debt equal to 1 billion and without taxes. Columns [D] and [E] correspond to a company with debt equal to 1 billion, a tax rate of 35% and different costs of debt. Column [F] corresponds to a company with a higher level of debt (2 billion) and a tax rate of 35%.

[10] The relationship between profit after tax (PAT) and profit before tax (PBT) is: PAT=PBT(1-T).

Table 1. Example of the valuation of six companies without growth

		[A] D=0 T=0%	[B] D=0 T=35%	[C] D=1,000 **T=0%** Kd=13%	[D] D=1,000 **T=35%** Kd=13%	[E] D=1,000 T=35% **Kd=14%**	[F] **D=2,000** T=35% Kd=14%
1	Margin	1,000	1,000	1,000	1,000	1,000	1,000
2	Interest	0	0	130	130	140	280
3	PBT	1,000	1,000	870	870	860	720
4	Taxes	0	350	0	304,5	301	252
5	PAT	1,000	650	870	565,5	559	468
6	+ depreciation	200	200	200	200	200	200
7	- Investment in fixed assets	-200	-200	-200	-200	-200	-200
8	**ECF**	**1,000**	**650**	**870**	**565,5**	**559**	**468**
9	**FCF**	**1,000**	**650**	**1,000**	**650**	**650**	**650**
10	**CCF**	**1,000**	**650**	**1,000**	**695,5**	**699**	**748**
11	Unlevered beta (ßu)	1	1	1	1	1	1
12	R_F	12%	12%	12%	12%	12%	12%
13	$(Rm - R_F)$ = market risk premium	8%	8%	8%	8%	8%	8%
14	Ku	20%	20%	20%	20%	20%	20%
15	**Vu**	**5,000**	**3,250**	**5,000**	**3,250**	**3,250**	**3,250**
16	D	0	0	1,000	1,000	1,000	2,000
17	Kd			13%	13%	14%	14%
18	Beta of debt (ßd)			0.125	0.125	0.25	0.25
19	VTS = DT	0	0	0	350	350	700
20	VTS + Vu	5,000	3,250	5,000	3,600	3,600	3,950
21	- D = **E1**	**5,000**	**3,250**	**4,000**	**2,600**	**2,600**	**1,950**
22	Levered beta ($ß_L$)	1	1	1.21875	1.21875	1.1875	1.5
23	Ke	20%	20%	21.75%	21.75%	21.50%	24%
24	**E2** = ECF / Ke	**5,000**	**3,250**	**4,000**	**2,600**	**2,600**	**1,950**
25	WACC	20%	20%	20%	18.06%	18.06%	16.46%
26	FCF / WACC	5,000	3,250	5,000	3,600	3,600	3,950
27	**E3** = (FCF / WACC) - D	**5,000**	**3,250**	**4,000**	**2,600**	**2,600**	**1,950**
28	$WACC_{BT}$	20%	20%	20%	19.32%	19.42%	18.94%
29	$CCF/WACC_{BT}$	5,000	3,250	5,000	3,600	3,600	3,950
30	**E4** = $(CCF / WACC_{BT})$ - D	**5,000**	**3,250**	**4,000**	**2,600**	**2,600**	**1,950**

Lines 1 to 5. The companies' income statements.

Lines 8, 9 and 10. Equity cash flow, free cash flow and capital cash flow.

Line 11. An unlevered beta βu (or assets' beta) equal to 1.0 is assumed.

Line 12. The risk-free rate is assumed to be equal to 12%.

Line 13. The market risk premium is taken to be 8%.

Line 14. With the above data, the required return to unlevered equity (Ku) is 20% in all cases.

Line 15. The value of the unlevered company (Vu = FCF/Ku) is 5 billion for the companies without taxes and 3.25 billion for the companies with a 35% tax rate. The difference (1.75 billion) is, logically, the present value of the taxes.

Lines 16 and 17. Company's debt and cost of debt.

Line 18. Beta corresponding to the cost of debt according to formula [15].

Line 19. Value of the tax shields due to interest payments, which, in this case (as it is a perpetuity), is DT.

Lines 20 and 21. They are the result of applying formula [9].

Line 22. Shows the beta of the equity according to formula [16].

Line 23. Shows the required return to equity according to formula [14].

Line 24. Calculation of the value of the equity is using formula [1].

Line 25. Weighted cost of equity and debt, calculated using the formula for WACC [5].

Lines 26 and 27. Calculation of the value of the equity is using formula [3].

Line 28. Weighted cost of equity and debt, calculated using the formula for $WACC_{BT}$ [8].

Lines 29 and 30. Calculation of the value of the equity is using formula [6].

Columns [B] and [D] show two very interesting points:

1. As they are perpetuities, according to formula [23], the risk of the equity cash flow is identical to the risk of the cash flow for the State (taxes).

2. Formula [9] proposes that the value of the levered company (D+E) is equal to value of the unlevered company (Vu) plus the value of the tax shields. Some authors argue that the value of the tax shields must be calculated by discounting the tax shields (interest x T = 130 x 0.35 = 45.5) at the required return to unlevered equity (Ku)[11]. This is not correct. In our example, this PV is 350 million, that is, 1,000 + 2,600 - 3,250 = 1,750 - 1,400. One can immediately see that 350 is not 45.5/0.2. In this case, 350 = 45.5/0.13, which explains why it seems that the correct discount rate is Kd[12]. Although ***in this case*** (perpetuities) the result is the same, we shall see further on that this is incorrect (except for perpetuities).

Table 2 highlights the most significant results of Table 1.

Table 2. Annual cash flows (million euros), discount rates and value of the company without growth

	WITHOUT TAXES		WITH TAXES (35%)	
	No debt	With debt	No debt	With debt
	D=0	D = 1,000	D=0	D = 1,000
ECF	1,000	870	650	565.5
Taxes	0	0	350	304.5
Debt flow (interest)	0	130	0	130
Total cash flow	1,000	1,000	1,000	1,000
Ke	20%	21.75%	20%	21.75%
Kd	—	13%	—	13%
K_{LT}	—	—	20%	21.75%
E = ECF/Ke	5,000	4,000	3,250	2,600
D = Debt flow/Kd	—	1,000	—	1,000
G = Taxes/K_{LT}	—	—	1,750	1,400
E+D+G	5,000	5,000	5,000	5,000
	[A]	[C]	[B]	[D]

(Columns of Table 1 to which these values correspond)

Other significant findings obtained from Table 1 include the following:

1. The required return to equity (Ke) decreases as the cost of debt increases, since the debt becomes an increasingly greater part of the business risk (Ku is constant and is not affected by leverage)[13]. Line 23, columns D and E.

2. The weighted cost of capital (WACC) does not depend on the cost of debt, but on the debt ratio and βu (not how βu is distributed between βd and βL). Line 25, columns D and E.

3. For the levered company with taxes, WACC is always less than Ku.

4. As the required return to debt is equal to the cost of debt, the equity value is independent of Kd: it depends on the debt value, but not on Kd. This does not mean that the debt's

[11] See, for example, Harris and Pringle (1985), Kaplan and Ruback (1995), Ruback (1995), and Tham and Vélez-Pareja (2001). All these papers are analyzed in Fernández (2004 and 2005).

[12] See, for example, Myers (1974) and Luehrman (1997). These papers are analyzed in Fernández (2004 and 2005).

[13] This is so because we are assuming that the debt's market value is the same as its nominal value. The required return to debt is equal to the cost of debt.

interest is irrelevant in real life. Obviously, if we think that the appropriate cost for the debt is 13% (thus, the debt has a value of 1,000 million) and the bank wants 14%, the shares' value will decrease because the debt's value is no longer 1,000 but 1,076.9 (140/0.13). However, the fact is that there is no formula that gives us the debt's risk from the business risk and the debt ratio. We only know that the business risk must be distributed between debt and equity in accordance with [16]. Consequently, the required return to debt has a certain degree of arbitrariness: it must be greater than R_F and less than Ku. Appendix 2 provides a formula for the required return to debt in the absence of leverage cost.

5. Formulae for when the debt's book value (N) is not the same as its market value (D). (r ≠ Kd)

N is the debt's book value (the money that the company has borrowed), **r** is the interest rate and **Nr** is the interest paid every year.

Kd is the required return to debt: a "reasonable" return that the bank or the bondholders must (or should) demand, in accordance with the company's risk and the size of the debt.

So far, we have assumed that the cost of debt (r) is equal to the return required by the market on that debt (Kd). However, if this is not so, the value of the debt (D) will no longer be the same as its nominal value (N). All the relationships calculated previously (assuming r = Kd) are valid for perpetuities irrespective of whether r and Kd are equal or not. It is sufficient to consider that in a perpetuity: D = N r / Kd

If r is equal to Kd, then D and N are equal.

[1], [2], [3] and all the formulae seen in this paper continue to be valid:

ECF = FCF - Nr (1 - T) = FCF - D Kd (1 - T)

6. Formula for adjusted present value taking into account the cost of leverage

We will assume now that the company loses value when it is levered. This loss of value is due to the "cost of leverage". Under this hypothesis, formula [9] becomes:

E+D = FCF/Ku + VTSNCL – cost of leverage

This formula indicates that the value of the levered company's debt today (D) plus that of its equity (E) is equal to the value of the equity (FCF/Ku) of the unlevered company plus the value of the tax shields with no-cost-of-leverage (VTSNCL) less the cost of leverage.

The cost of leverage includes a series of factors: the greater likelihood of bankruptcy or voluntary reorganization, information problems, reputation, difficulty in gaining access to growth opportunities, differential costs in security issues, and other associated considerations. These costs increase with higher debt levels.

6.1. Impact on the valuation of using the simplified formulae for the levered beta

Two ways of quantifying the cost of leverage are to use the simplified formulae for calculating the levered beta[14] ([27] and [28])instead of [17]:

[27] $\beta^*_L = \beta_U [D + E^*] / E^*$. **[28]** $\beta'_L = \beta_U [D (1 - T) + E'] / E'$

[17] $$\beta_L = \frac{\beta_U [E + D(1-T)] - \beta_d D(1-T)}{E}$$

If these simplified formulae are used, the levered betas obtained (β^*_L and β'_L) will be greater than those obtained using the full formula [17].

In addition, the value of the equity (E* or E') will be less than that obtained earlier (E) because the required return to equity now (Ke* or Ke') is greater than that used previously (Ke). Logically, the weighted cost of debt and equity now (WACC* or WACC') is greater than that used earlier (WACC).

In the example: $\beta_L = 1.375$; $\beta'_L = 1.659$; $\beta^*_L = 2.333$

E = 1.500; E' = 1.365; E* = 1.125. Ke = 23%; Ke' = 25.275%; Ke* = 30.667%.

Observe that: $E^* < E' < E$ and $Ke^* > Ke' > Ke$

With these simplifications, we introduce cost of leverage in the valuation: in formula [9], we must add a term CL that represents the cost of leverage.

[9*] $E^* = FCF / Ku – D (1 - T) - CL^*$ **[9']** $E' = FCF / Ku – D (1 - T) - CL'$

$CL^* = E – E^*$ $CL' = E – E'$

[4] continues to be valid: ECF = FCF - D Kd (1 - T)

In the example: WACC = 16%; WACC' = 16.754%; WACC* = 18.286%. CL* = 375; CL' = 135;

Using these formulae, we obtain the following relationships:

[29] $CL' = E – E' = D (Kd - R_F) (1 - T) / Ku$

[30] $CL^* = E – E^* = [D (Kd - R_F) (1 - T) + DT (Ku - R_F)] / Ku$

6.2. The simplified formulae as a leverage-induced reduction of the FCF

The simplified formulae can be viewed as a reduction of the expected FCF (due to the constraints and restrictions caused by the debt) instead of as an increase in the required return to equity. In formula [9], the FCF is independent of leverage (having the size of D).

[14] The theory we call β' here corresponds to Damodaran (1994) and the theory that we call β* here corresponds to the practitioners method.

If we use formula [28]: β'L = βu [D (1 - T) + E'] / E', we can consider that the value E' is obtained from discounting another smaller cash flow (FCF´) at the rate of the full formula:

$$E' = \frac{FCF}{Ku} - D + \frac{D\left[Ku\,T - (1-T)\left(Kd - R_F\right)\right]}{Ku} = \frac{FCF'}{Ku} - D(1-T) = \frac{ECF'}{Ke}$$

so

[31] $(FCF - FCF') = D\left[(1-T)(Kd - R_F)\right] = ECF - ECF'$

This means that when we use the simplified formula [28], we are considering that the free cash flow and the equity cash flow are reduced by the quantity D (1 - T) (Kd - R_F).

Likewise, if we use formula [27]: β^*_L = βu [D + E*] / E*, we can consider that the value E* is obtained from discounting another smaller cash flow (FCF*) at rate of the full formula:

$$E^* = \frac{FCF}{Ku} - D + \frac{D\left[R_F - Kd(1-T)\right]}{Ku} = \frac{FCF^*}{Ku} - D(1-T) = \frac{ECF^*}{Ke}$$

[32] $(FCF - FCF^*) = D\left[T(Ku - R_F) + (1-T)(Kd - R_F)\right] = ECF - ECF^*$

This means that when we use the simplified formula [27], we are considering that the free cash flow (and the equity cash flow) are reduced by D [T(Ku - R_F) + (1-T) (Kd - R_F)].

6.3. The simplified formulae as a leverage-induced increase in the business risk (Ku)

Another way of viewing the impact of using the abbreviated formula [28] is to assume that what the formula proposes is that the business risk increases with leverage. In order to measure this increase, we call βu the business's beta for each level of leverage. Using formula [28] with βu´ instead of βu, upon performing the algebraic operations, it is seen that:

[33] βu' = βu + β_d D (1-T) / [D (1 - T) + E']

Likewise, the impact of using the simplified formula [27] β^*_L = βu [D + E*] / E* can be measured by assuming that the formula proposes that the business risk (which we will quantify as βu*) increases with leverage. Using formula [1] with βu* instead of βu, upon performing the algebraic operations, it is seen that:

[34] βu* = βu + [β_d D (1-T) + βu TD] / [D (1 - T) + E*]

It can also be seen that:

[35] $$Ku' = Ku + (Kd - R_F)\frac{D(1-T)}{E' + D(1-T)}$$

[36] $$Ku^* = Ku + (Kd - R_F)\frac{D(1-T)}{E^* + D(1-T)} + (Ku - R_F)\frac{DT}{E^* + D(1-T)}$$

6.4. The simplified formulae as a probability of bankruptcy

This model includes the possibility that the company goes bankrupt and ceases to generate cash flows:

ECF_{t+1} =	ECF_t	with a probability $p_c = 1 - p_q$
	$0 = E_{t+1}$	with a probability p_q

In this case, the equity value at t = 0 is:

[37] $E^* = ECF\ (1 - p_q^*) / (Ke + p_q^*)$

It can be seen immediately that, if E = ECF/Ke, $p_q^* = Ke\ (E - E^*) / (E^* + E\ Ke) = (ECF - E^*\ Ke) / (E^* + ECF)$

6.5. Impact of the simplified formulae on the required return to equity

Using the simplified formulae changes the relationship between Ke and Ku. Without costs of leverage, that is, using formula [17], the relationship is [16]:

[16] $Ke = Ku + [D\ (1-T)/E]\ (Ku – Kd)$

Using formula [27], the relationship is:

[38] $Ke^* = Ku + (D/E^*)\ (Ku – R_F)$

Using formula [28], the relationship is:

[39] $Ke' = Ku + [D\ (1-T)/E']\ (Ku – R_F)$

7. Valuing companies using discounted cash flows. Constant growth

In the previous sections, we defined the concepts and parameters used to value companies without growth and infinite life (perpetuities). In this section, we will discuss the valuation of companies with constant growth.

Initially, we assume that the debt’s market value is the same as its book value. Section 8.2 addresses the case of mismatch between the debt’s book value (N) and its market value (D), which is very common in practical reality. Section 8.3 analyzes the impact on the valuation of using simplified betas.

Now, we will assume that the cash flows generated by the company grow indefinitely at a constant annual rate g > 0. This implies that the debt to equity (D/E) and the working capital requirements to net fixed assets (WCR/NFA) ratios remain constant, or, to put it another way, debt, equity, WCR and NFA grow at the same rate g as the cash flows generated by the company.

In the case of perpetuities, as FCF, ECF and CCF were constant, it was not important to determine the period during which the various cash flows used in the valuation formula were generated. In the case of companies with constant growth, by contrast, it is necessary to consider the period: a period's expected cash flow is equal to the sum of the previous period's cash flow plus the growth g. For example, $FCF_1 = FCF_0 (1+g)$.

8. Company valuation formulae. Constant growth

With constant growth (g), the discounted cash flow valuation formulae are:

[1g] $E = ECF1 / (Ke - g)$

[2g] $$E + D = \frac{ECF_1}{Ke-g} + \frac{CFd_1}{Kd-g} = \frac{ECF_1}{Ke-g} + \frac{D\ Kd - gD}{Kd-g}$$

[3g] $E + D = FCF1 / (WACC - g)$

[6g] $E + D = CCF1 / (WACC_{BT} - g)$

[9g] $E + D = FCF_1 / (Ku - g) + VTSNCL - \text{Cost of leverage}$

The formula that relates FCF and ECF is:

[4g] $ECF_1 = FCF_1 - D_0 [Kd (1 - T) - g]$

because $ECF_1 = FCF_1 - I_1 (1 - T) + \Delta D_1$; $I_1 = D_0 Kd$; and $\Delta D_1 = g D_0$

The formula that relates CCF with ECF and FCF is:

[7g] $CCF_1 = ECF_1 + D_0 (Kd - g) = FCF_1 - D_0 Kd T$

Although it is obvious, it is useful to point out that the debt's value at t = 0 (D_0) is[15]

$$D_0 = \frac{(I - \Delta D)_1}{Kd-g} = \frac{KdD_0 - gD_0}{Kd-g} = D_0$$

8.1. Relationships obtained from the formulae

As seen in sections 2.2 and 2.3, it is possible to infer the same relationships by pairing formulae [1g] to [9g] and proceeding on the basis that the results given must be equal. For the moment, we will assume that the cost of leverage is zero.

---

15 Note that we are assuming that the debt's market value is equal to its nominal or book value.

As [2g] must be equal to [3g], using [4g], we obtain the definition of WACC [5]
As [2g] must be equal to [6g], using [7g], we obtain the definition of $WACC_{BT}$ [8]
As [3g] must be equal to [9g], without cost of leverage, it follows:

(E+D) (WACC-g) = (E+D-VTS) (Ku-g), so[16]: VTS = (E+D) (Ku-WACC) / (Ku-g)

Substituting in this equation the expression for WACC [5] and taking into account [12], we obtain:

[10g] VTS = D T Ku / (Ku-g)

We would point out again that this expression is not the PV of a single cash flow, but the difference between the present values of two cash flows, each with a different risk: the taxes of the company without debt and the taxes of the company with debt.

One conclusion that is drawn from the above expressions is that the debt cash flow and the equity cash flow (and, therefore, the tax cash flow) depend on Kd, but the value of the debt D (which has been preset and is assumed to be equal to its nominal value), the value of the equity E and, therefore, the value of the taxes *do not depend* on Kd[17].

If we were to discount the tax shields due to interest payments at the rate Kd, this would give:

VTS = D Kd T /(Kd – g) , which does depend on Kd.

Consequently, ***the VTS is not the present value of the tax shields due to interest payments (D Kd T) at the rate Kd.*** The reason is that the value of the tax shields is not the PV of a cash flow (D Kd T, which grows at a rate g), but the difference between the present values of two cash flows with a different risk: the PV of the taxes of the company without debt at the rate K_{TU} and the PV of the taxes of the company with debt at the rate K_{TL}.

8.2. Formulae when the debt's book value (N) is not equal to its market value (D)

N is the book value of debt (the money that the company has borrowed), **r** is the interest rate and **Nr** is the annual interest payment.

Kd is the required return to debt: a "reasonable" return that the bondholders or the bank must (or should) demand, in accordance with the company's risk and the size of the debt. Therefore, Kd D is the interest which, from the "reasonable" viewpoint, the company should pay.

Until now, we have assumed that r = Kd, but if this is not so, the debt's market value (D) will not be equal to its nominal value (N).

If the debt grows annually $\Delta N_1 = g N_0$, then:

[40] D = N (r – g)/(Kd – g)

So: D Kd - Nr = g (D - N).

[16] The same result could be obtained by making [2] and [5] equal (using [6]).

[17] This is because we are assuming that the debt's market value (D) is equal to its book value (N).

The relationship between ECF and FCF is:

[**41**] $ECF = FCF - Nr (1-T) + gN = FCF - D (Kd - g) + N r T$

As can be seen, when $r \neq Kd$, the relationship between ECF and FCF is not equal to the relationship when $r = Kd$.

Substituting [41] and [1g] in [3g]:

$$E + D = \frac{ECF + D(Kd - g) - NrT}{WACC - g} = \frac{E(Ke - g) + D(Kd - g) - NrT}{WACC - g}$$

Upon performing algebraic operations, we obtain:

[42] $$WACC = \frac{E\,Ke + D\,Kd - Nr\,T}{E + D}$$

It can also be shown that the expression for calculating the VTS is:

[43] $$VTS = \frac{D\,T\,Ku + T\,[Nr - D\,Kd]}{Ku - g}$$

As we have already seen that $D\,Kd - D\,g = N\,r - N\,g$, it is clear that: $N\,r - D\,Kd = g\,(N - D)$

Substituting, this gives:

$$VTS = D\,T + \frac{T\,g\,N}{Ku - g} = \frac{D\,T\,(Ku - g) + T\,g\,N}{Ku - g}$$

8.3. Impact of the use of the simplified formulae

$\beta^*_L = \beta_U [D + E^*] / E^*$ and $\beta'_L = \beta_U [D (1 - T) + E'] / E'$

If these simplified formulae are used, the levered beta (β_L*) will be greater than that obtained using the full formula [19.17]:

$\beta_L = \beta_U + D (1 - T) [\beta_U - \beta_d] / E$

In addition, the value of the equity (E* or E') will be less than that obtained previously (E) because the required return to equity now (Ke* or Ke') is greater than that used previously (Ke). Logically, the weighted cost of debt and equity now (WACC') is greater than that used previously (WACC).

With these simplifications, we introduce cost of leverage in the valuation: in formula [5], we must add the term CL, which represents the cost of leverage: increase of risk and/or decrease in FCF when the debt ratio increases.

Using the same methodology followed in the section on perpetuities, we can obtain the different expressions for equity value that are obtained using the full formula (E) or the abbreviated formulae (E', E*). For a company whose FCF grows uniformly at the annual rate g, they are[18]:

[29g] $$CL' = E - E' = \frac{D\,(1 - T)(Kd - R_F)}{Ku - g}$$

[30g] $$CL^* = E - E^* = \frac{D(1 - T)(Kd - R_F)}{Ku - g} + \frac{DT(Ku - R_F)}{Ku - g}$$

9. Examples of companies with constant growth

Table 3 shows the balance sheet, income statement, and cash flows of a company with a growth of 5% in all the parameters except net fixed assets, which remain constant.

Table 3. Balance sheet, income statement and cash flows of a company that grows at 5%. The net fixed assets remain constant. T = 35%

		0	1	2	3	4
1	Cash and banks	100	105	110.25	115.76	121.55
2	Accounts receivable	900	945	992.25	1,041.86	1,093.96
3	Stocks	240	252	264.60	277.83	291.72
4	Gross fixed assets	1,200	1,410	1,630.50	1,862.03	2,105.13
5	- cum. depreciation	200	410	630.50	862.03	1,105.13
6	Net fixed assets	1,000	1,000	1,000	1,000	1,000
7	**TOTAL ASSETS**	**2,240**	**2,302**	**2,367.10**	**2,435.46**	**2,507.23**
8	Accounts payable	240	252	264.60	277.83	291,72
9	Debt	500	525	551.25	578.81	607.75
10	Equity (book value)	1,500	1,525	1,551.25	1,578.81	1,607.75
11	**TOTAL LIABILITIES**	**2,240**	**2,302**	**2,367.10**	**2,435.46**	**2,507.23**
	Income statement					
12	Sales	3,000	3,150	3,307.50	3,472.88	3,646.52
13	Cost of sales	1,200	1,260	1,323.00	1,389.15	1,458.61
14	General expenses	600	630	661.50	694.58	729.30
15	Depreciation	200	210	220.50	231.53	243.10
16	Margin	1,000	1,050	1,102.50	1,157.63	1,215.51
17	Interest	75	75	78.75	82.69	86.82
18	PBT	925	975	1,023.75	1,074.94	1,128.68
19	Taxes	323.75	341.25	358.31	376.23	395.04
20	PAT	601.25	633.75	665.44	698.71	733.64
21	+ Depreciation		210	220.50	231.53	243.10
22	+ Δ Debt		25	26.25	27.56	28.94
23	- Δ WCR		-50	-52.50	-55.13	-57.88
24	- Investments		-210	-220.50	-231,53	-243.10
25	ECF = Dividends		608.75	639.19	671,15	704.70
26	FCF		632.50	664.13	697.33	732.20
27	CCF		658.75	691.69	726.27	762.59
28	Debt cash flow		50.00	52.50	55.13	57.88

Lines 1 to 11 show the forecasts for the company's balance sheet for the next 5 years. Lines 12 to 20 show the forecast income statements.

Lines 21 to 25 show the calculation of the equity cash flow in each year. Line 26 shows each year's free cash flow. Line 27 shows each year's capital cash flow. Line 28 shows each year's debt cash flow.

The growth of the equity cash flow, free cash flow, capital cash flow, and debt cash flow is 5% per annum.

[18] Note that in all cases we are considering the same debt (D) and the same cost (Kd).

Table 4 shows the valuation of the company with a growth of 5% in all the parameters except net fixed assets, which remain constant. Line 1 shows the beta for the unlevered company (which is equal to the net assets' beta = βu), which has been assumed to be equal to 1. Line 2 shows the risk-free rate, which has been assumed to be 12%. Line 3 shows the market risk premium, which has been assumed to be 8%. These results are used to calculate line 4, which gives Ku = 20%.

Table 4. Valuation of a company that grows at 5%.
The net fixed assets are constant. T = 35%

			0	**1**	**2**	**3**	**4**
1		Beta U	1	1	1	1	1
2		R_F	12%	12%	12%	12%	12%
3		R_M - R_F	8%	8%	8%	8%	8%
4		Ku	20%	20%	20%	20%	20%
5		Vu = FCF/(Ku - g)	4,216.67	4,427.50	4,648.88	4,881.32	5,125.38
	WITHOUT TAXES						
6		FCF WITHOUT TAXES		1,000.00	1,050.00	1,102.50	1,157.63
7		Vu without taxes	6,666.67	7,000.00	7,350.00	7,750	8,103.38
	WITH TAXES						
8		Kd	***15%***	***15%***	***15%***	***15%***	***15%***
9		Beta d	0.375	0.375	0.375	0.375	0.375
10		DTKu/(Ku-g) = VTS	233.33	245.00	257.25	270.11	283.62
11		VTS + Vu	4,450.00	4,672.50	4,906.13	5,151.43	5,409.00
12	- D =	**E 1**	**3,950**	**4,148**	**4,355**	**4,573**	**4,801**
13		Beta E	1.05142	1.05142	1.05142	1.05142	1.05142
14		Ke	20.41%	20.41%	20.41%	20.41%	20.41%
15		**E 2 = ECF / (Ke-g)**	**3,950**	**4,148**	**4,355**	**4,573**	**4,801**
16		WACC	19.213%	19.213%	19.213%	19.213%	19.213%
17	D + E =	FCF / (WACC-g)	4,450.00	4,672.50	4,906.13	5,151.43	5,409.00
18	- D =	**E 3**	**3,950**	**4,148**	**4,355**	**4,573**	**4,801**
19		$WACC_{BT}$	19.803%	19.803%	19.803%	19.803%	19.803%
20	D + E =	CCF / ($WACC_{BT}$-g)	4,450.00	4,672.50	4,906.13	5,151.43	5,409.00
21	- D =	**E 4**	**3,950**	**4,148**	**4,355**	**4,573**	**4,801**

Line 5 shows the value of the unlevered company Vu by discounting the future free cash flows at the rate Ku.

Lines 6 and 7 show what the company's free cash flow would be if there were no taxes, and what Vu would be if there were no taxes.

Line 8 shows the cost of debt, which has been assumed to be 15%. Line 9 is the debt's beta ($ß_d$) corresponding to its cost (15%), which gives 0.375.

Line 10 shows the value of the tax shields due to interest payments. Line 11 is the application of formula [9].

Line 12 is obtained by subtracting the value of the debt from line 11, obtaining the value of the equity.

Line 13 shows the equity's beta ($ß_L$). Line 14 shows the required return to equity corresponding to the beta in the previous line. Line 15 is the result of using formula [1]. It is equal to line 12.

Line 16 shows the weighted average cost of capital (WACC). Line 17 shows the present value of the free cash flow discounted at the WACC. Line 18 shows the value of the equity according to formula [3], which is also equal to lines 12 and 15.

Line 19 shows the weighted cost of equity and debt before tax ($WACC_{BT}$). Line 20 shows the present value of the capital cash flow discounted at the $WACC_{BT}$. Line 21 shows the value of the equity according to formula [4], which is also equal to lines 12, 15 and 18.

It is important to realize that although the cash flows in Tables 3 and 4 grow at 5%, the economic profit and the EVA do not grow at 5%. The reason is that, in these tables, the net fixed assets remain constant (investments = depreciation).

Table 5 highlights the most important results obtained from Tables 3 and 4.

Table 5. Cash flows in year 1, discount rates and value of the company with an annual growth = 5%

	WITHOUT TAXES		WITH TAXES	
	Without debt D=0	**With debt D = 500**	**Without debt D=0**	**With debt D = 500**
ECF	1,000	950	632.5	608.75
Taxes	—	—	367.5	341.25
Debt cash flow	**—**	**50**	**—**	**50**
Ke	20%	20.40%	20%	20.41%
Kd	—	15%	—	15%
K_{TL}	—	—	20%	20.39%[19]
E = ECF/(Ke-g)	6,667	6,167	4,217	3,950
G = Taxes/(K_{TL}-g)	—	—	2,450	2,217
D = Debt cash flow/(Kd-g)	—	500	—	500
SUM	**6,667**	**6,667**	**6,667**	**6,667**

It is important to point out that the tax risk is different from the equity cash flow risk. The risk of both flows will be identical only if the sum of tax and equity cash flow is equal to PBT. This only happens if the ECF is equal to PAT, as tax amounts to 35% of the PBT.

In Table 3 (year 1, D=500, T=35%), the equity cash flow (608.75) is less than the PAT (633.75). Consequently, tax has less risk than the equity cash flow.

10. Tax risk and VTS with constant growth

Formula [18] continues to be valid when a similar development (without leverage costs) to that of section 3 for perpetuities is performed:

[18] $Vu_t + Gu_t = E_t + D_t + GL_t$

The value of the tax shields (VTS) is:

[19] $VTS_t = Gu_t - GL_t$

In a company with constant growth and without debt, the relationship between taxes and profit before tax is: $Taxes_U$ = T PBTu.

The relationship between taxes and free cash flow is different from that obtained for perpetuities:

[20]g $Taxes_U = T [FCF + g(WCR + NFA)] / (1-T) = T [FCF + g(Ebv+D)] / (1-T)$

[19] This is obtained from: 341.25 / (K_{TL} – 0.05) = 2.217

WCR is the net working capital requirements. NFA is the net fixed assets. Ebv is the equity book value.

The present value of taxes in the unlevered company is:

[22]g $G_U = Taxes_U \; / (K_{TU} - g)$

In a levered company with constant growth, the relationship between taxes and equity cash flow is different from that obtained for perpetuities:

[23]g $Taxes_L = T (ECF + g\ Ebv) / (1-T)$.

The present value of taxes in the levered company is:

[25]g $G_L = Taxes_L \; / (K_{TL} - g)$

The increase in the value of the company due to the use of debt ***is not*** the present value of the tax shields due to the payment of interest but the difference between G_U and G_L, which are the present values of two cash flows with a different risk:

[26]g $VTS_t = Gu_t - GL_t = [Taxes_U \; / (KT_U - g)] - [Taxes_L \; / (KT_L - g)]$

Assuming that there are no costs of leverage, the following is obtained:

[10]g $VTS_t = D\ T\ Ku / (Ku - g)$

11. Valuation of companies by discounted cash flow. General case

In the previous sections, valuation parameters and concepts have been defined and applied to two specific cases: perpetuities and constant growth. Now, the subject will be discussed on a general level, i.e. without any predefined evolution of the cash flows over the years. In addition, the study period may be finite.

In the course of the following sections, it is shown:

1. The tax shields due to interest payments (VTS) must ***not*** be discounted (as many authors propose) at the rate Ke (required return to equity) nor at the rate Kd (required return to debt).

2. The value of the tax shields due to interest payments (without costs of leverage) is equal to the PV of the tax shields that would exist if the debt had a cost equal to Ku. That is because this PV is not exactly the present value of a cash flow, but the difference between two present values: that of the flow of taxes paid by the unlevered company and that of the flow of taxes paid by the levered company (flows with different risk).

VTS = PV [D Ku T; Ku]

3. Expression of the WACC when the debt's book value is not equal to its "market" value.

4. Expression of the VTS when the debt's book value is not equal to its "market" value.

5. The impact on the valuation of using the simplified formulae for the levered beta.

12. Company valuation formulae. General case

There follow four formulae for company valuation using discounted cash flows for a general case. By this we mean that the cash flows generated by the company may grow (or contract) at a different rate each year, and thus, all of the company's parameters can vary from year to year, such as, for example, the level of leverage, the WCR or the net fixed assets.

[44] $$E_0 = \sum_{t=1}^{\infty} \left[ECF_t \Big/ \prod_{1}^{t} (1 + Ke_t) \right] = PV\ (Ke;\ ECF)$$

Let us now see the other expressions. The formula which relates the FCF to the company's value is:

[45] $\mathbf{E_0 + D_0 = PV\ (WACC;\ FCF)}$

The formula that relates the CCF to the company's value is:

[46] $\mathbf{E_0 + D_0 = PV\ (WACC_{BT};\ CCF)}$

Other relevant expressions are:

[47] $E_1 = Eo\ (1+Ke_1) - ECF_1$

[48] $D_1 + E_1 = (Do+Eo)\ (1+WACC_1) - FCF_1$

[49] $D_1 + E_1 = (Do+Eo)\ (1+WACC_{BT1}) - CCF_1$

We can also calculate the value of $D_0 + E_0$ from the value of the unlevered company:

[50] $\mathbf{E_0 + D_0 = PV\ (Ku;\ FCF) + VTSNCL - cost\ of\ leverage}$

13. Relationships obtained from the formulae. General case

There follows a number of important relationships that can be inferred by pairing formulae [44], [45], [46], and [50], and taking into consideration that the results they give must be equal.

If r = Kd and Cost of leverage = 0

[51] $ECFt = FCFt + \Delta Dt - I_t\ (1 - T)$

[52] $CCF_t = ECFt - \Delta Dt + I_t$ $\Delta Dt = Dt - Dt\text{-}1$ $It = D_{t-1}\ Kdt$

[53] $$D_0 = \sum_{t=1}^{\infty} \frac{D_{t-1}\ Kd_t - (D_t - D_{t-1})}{\prod_{1}^{t} (1 + Kd_t)}$$

[54] $WACC_t = \frac{E_{t-1}\, Ke_t + D_{t-1}\, Kd_t\, (1-T)}{(E_{t-1} + D_{t-1})}$ **[55]** $WACC_{BTt} = \frac{E_{t-1}\, Ke_t + D_{t-1}\, Kd_t}{(E_{t-1} + D_{t-1})}$

As:

[56] $Ku_t = \frac{E_{t-1}\, Ke_t + D_{t-1}\, Kd_t\, (1-T)}{E_{t-1} + D_{t-1}(1-T)}$ which is equivalent to [12], gives

[57] $WACC_t = \frac{E_{t-1} + D_{t-1}\, (1-T)}{(E_{t-1} + D_{t-1})} Ku_t$ **[58]** $VTS_0 = \sum_{t=1}^{\infty} \frac{D_{t-1}\, Ku_t\, T}{\prod_{1}^{t} (1 + Ku_t)}$

The following identities must be remembered:

[18] $Vu_t + GU_t = E_t + D_t + GL_t$

$Vu_t\ Ku_{t+1} + GU_t\ K_{TU\ t+1} = E_t\ Ke_{\ t+1} + D_t\ Kd_{\ t+1} + GL_t\ K_{TL\ t+1}$

[19] $VTS_t = GU_t - GL_t = E_t + D_t - Vu_t$

14. An example of company valuation

Table 6 shows the previous balance sheets of the company Font, Inc.

Table 6. Forecast balance sheets for Font, Inc.

		0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
1	Cash	100	120	140	160	180	200	210	220	230.0	240.0	252.0
2	Accounts receivable	900	960	1,020	1,080	1,140	1,200	1,260	1,320	1,380.0	1,449.0	1,521.5
3	Stocks	300	320	340	360	380	400	420	440	460.0	483.0	507.2
4	Gross fixed assets	1,500	1,800	2,700	3,100	3,300	3,500	3,900	4,204	4,523.2	4,858.4	5,210.3
5	- cum. depreciation	200	550	900	1,300	1,800	2,100	2,380	2,684	3,003.2	3,338.4	3,690.3
6	Net fixed assets	1300	1,250	1,800	1,800	1,500	1,400	1,520	1,520	1,520.0	1,520.0	1,520.0
7	**TOTAL ASSETS**	**2,600**	**2,650**	**3,300**	**3,400**	**3,200**	**3,200**	**3,410**	**3,500**	**3,590.0**	**3,692.0**	**3,800.6**
8	Accounts payable	300	320	340	360	380	400	420	440	460.0	483.0	507.2
9	Debt	1,800	1,800	2,300	2,300	2,050	1,800	1,700	1,450	1,200.0	1,000.0	1,050.0
10	Equity	500	530	660	740	770	1,000	1,290	1,610	1,930.0	2,209.0	2,243.5
11	**TOTAL**	**2,600**	**2,650**	**3,300**	**3,400**	**3,200**	**3,200**	**3,410**	**3,500**	**3,590.0**	**3,692.0**	**3,800.6**

Table 7 shows the income statements and the cash flows.

Table 7. Forecast income statements and cash flows for Font, Inc.

		1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*
14	Sales	3,200	3,400	3,600	3,800	4,000	4,200	4,400	4,600	4,830	5,071.50	5,325.08
15	Cost of sales	1,600	1,700	1,800	1,900	2,000	2,100	2,200	2,300	2,415	2,535.75	2,662.54
16	General expenses	800	850	900	950	1,000	1,050	1,100	1,150	1,207.50	1,267.88	1,331.27
17	Depreciation	350	350	400	500	300	280	304	319.20	335.16	351.92	369.51
18	Margin	450	500	500	450	700	770	796	830.80	872.34	915.96	961.75
19	Interest	270	270	345	345	307.50	270	255	250	180	150	158
20	PBT	180	230	155	105	392.50	500	541	613.30	692.34	765.96	804.25
21	Tax	63	80.5	54.25	36.75	137.38	175	189.35	214.66	242.32	268.08	281.49
22	**PAT**	**117**	**149.5**	**100.75**	**68.25**	**255.13**	**325**	**351.65**	**398.65**	**450.02**	**497.87**	**522.77**
23	+ Depreciation	350	350	400	500	300	280	304	319.20	335.16	351.92	369.51
24	+ Δ Debt	0	500	0	-250	-250	-100	-250	-250	-200	50	52.50
25	- Δ WCR	-80	-80	-80	-80	-80	-70	-70	-70	-79	-84.45	-88.67
26	- Investments	-300	-900	-400	-200	-200	-400	-304	-319.20	-335.16	-351.92	-369.51
27	**ECF= Dividends**	**87**	**19.5**	**20.75**	**38.25**	**25.13**	**35**	**31.65**	**78.65**	**171.02**	**463.42**	**486.59**
28	**FCF**	**262.5**	**-305**	**245**	**512.5**	**475**	**310.5**	**447.40**	**470.02**	**488.02**	**510.92**	**536.47**

Table 8 assumes that the cost of leverage is zero. It shows the valuation by all four methods for a company that is growing (but not at a constant rate) up to year 9. After year 9, a constant growth of 5% has been forecasted. The cash flows grow at 5% from year 11 onwards. The cash flows of year 10 are not 5% greater than those of year 9.

For this general case, too, it is seen that all our valuation formulae ([44], [45], [46] and [50]) give the same value for the company's equity: at t = 0, it is 506 million euros (see lines 43, 46, 50 and 53).

It can also be seen that:

1) The value of the tax shields due to interest payments is 626.72 million (line 41).

2) It would be mistaken to calculate the value of the tax shields by discounting DTKd at the debt interest rate (15%), as that would give 622 million.

Table 8. Valuation of Font, Inc.

		0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
35	**Ku**	**20%**	**20%**	**20%**	**20%**	**20%**	**20%**	**20%**	**20%**	**20%**	**20%**	**20%**
36	Vu	1,679.6	1,753.1	2,408.7	2,645.4	2,662.0	2,719.4	2,952.8	3,096.0	3,245.1	3,406.1	3,576.5
39	**Kd**	15%	15%	15%	15%	15%	15%	15%	15%	15%	15%	15%
40	Beta d	0.3750	0.3750	0.3750	0.3750	0.3750	0.3750	0.3750	0.3750	0.3750	0.3750	0.3750
41	VTS	626.72	626.06	625.28	589.33	546.20	511.94	488.33	466.99	458.89	466.67	490.00
42	VTS + Vu	2,306.37	2,379.14	3,033.97	3,234.76	3,208.22	3,231.36	3,441.13	3,562.96	3,704.03	3,872.81	4,066.45
43	**-D=E1**	**506**	**579**	**734**	**935**	**1,158**	**1,431**	**1,741**	**2,113**	**2,504**	**2,873**	**3,016**
44	Beta E	2.4441	2.2626	2.2730	1.9996	1.7190	1.5109	1.3967	1.2788	1.1947	1.1414	1.1414
45	**Ke**	**31.55%**	**30.10%**	**30.18%**	**28.00%**	**25.75%**	**24.09%**	**23.17%**	**22.23%**	**21.56%**	**21.13%**	**21.13%**
46	**E 2 = PV(Ke;ECF)**	**506**	**579**	**734**	**935**	**1,158**	**1,431**	**1,741**	**2,113**	**2,504**	**2,873**	**3,016**
47	$E_t = E_{t-1}(1+Ke)$ - ECF	506	579	734	935	1,158	1,431	1,741	2,113	2,504	2,873	3,016
48	**WACC**	**14.54%**	**14.70%**	**14.69%**	**15.02%**	**15.53%**	**16.10%**	**16.54%**	**15%**	**73%**	**18.19%**	**18.19%**
49	PV(WACC;FCF)	2,306.37	2,379.14	3,033.97	3,234.76	3,208.22	3,231.36	3,441.13	3,562.96	3,704.03	3,872.81	4,066.45
50	**- D = E 3**	**506**	**579**	**734**	**935**	**1,158**	**1,431**	**1,741**	**2,113**	**2,504**	**2,873**	**3,016**
51	**$WACC_{BT}$**	**18.63%**	**18.68%**	**18.67%**	**18.76%**	**18.88%**	**19.03%**	**19.14%**	**19.29%**	**19.43%**	**19.55%**	**19.55%**
52	PV($WACC_{BT}$; CCF)	2,306.37	2,379.14	3,033.97	3,234.76	3,208.22	3,231.36	3,441.13	3,562.96	3,704.03	3,872.81	4,066.45
53	**- D = E 4**	**506**	**579**	**734**	**935**	**1,158**	**1,431**	**1,741**	**2,113**	**2,504**	**2,873**	**3,016**

The lines of Tables 6, 7 and 8 have the following meanings:

Lines 1 to 11 show the forecast balance sheets for the company over the next 10 years.

Lines 14 to 22 show the forecast income statements.

Lines 23 to 27 show the calculation of each year's equity cash flow.

Line 28 shows each year's free cash flow.

Line 35 shows Ku = 20%. This result comes from a risk-free rate of 12%, a market risk premium of 8%, and a beta for the unlevered company equal to 1.

Line 36 shows the value of the unlevered company (Vu), discounting the future free cash flows at the rate Ku at t = 0 (now), giving Vu = 1,679.65.

Lines 37 and 38 show what would be the company's free cash flow if there were no taxes and what would be Vu with no taxes. If there were no taxes, at t = 0 Vu = 2,913

Line 39 shows the cost of the debt, which has been assumed to be 15%.

Line 40 shows the debt's beta corresponding to its cost, which gives 0.375.

Line 41 shows the value of the tax shields due to interest payments, which at t = 0 is 626.72.

Line 42 is the application of formula [50]. At t = 0, it gives D + E = 1,679.65 + 626.72 = 2,306.37.

Line 43 is the result of subtracting the value of the debt from line 42. At t = 0, the value of the equity is 506 million.

Line 44 shows the equity's beta, using formula [17].

Line 45 shows the required return to equity corresponding to the beta in the previous line.

Line 46 is the result of using formula [44]. This formula, too, finds that the value of the equity at t = 0 is 506 million. Line 47 shows the evolution of the equity's value according to formula [47]. Note that line 47 is the same as line 46.

Line 48 shows the weighted cost of equity and debt after tax, WACC, according to formula [54].

Line 49 shows the present value of the free cash flow discounted at the WACC.

Line 50 shows the value of the equity according to formula [45], which is also found to be 506 million.

Line 51 shows the weighted cost of equity and debt before tax $WACC_{BT}$, according to formula [55].

Line 52 shows the present value of the capital cash flow discounted at the $WACC_{BT}$.

Line 53 shows the value of the equity according to formula [46], which is also found to be 506 million.

Table 9 shows a sensitivity analysis of the equity after making changes in certain parameters.

Table 9. Sensitivity analysis of the value of the equity at t = 0 (in million)

Value of Font, Inc.'s equity in Table 3	**506**
Tax rate = 30% (instead of 35%)	594
Risk-free rate (R_F) = 11% (instead of 12%)	653
Market (P_M) = 7% (instead of 8%)	653
βu = 0.9 (instead of 1.0)	622
Residual growth (after year 9) = 6% (instead of 5%)	546

15. Valuation formulae when the debt's book value (N) and its market value (D) are not equal

Our starting point is:

[59] $$D_0 = \sum_{t=1}^{\infty} \frac{N_{t-1}\, r_t - (N_t - N_{t-1})}{\prod_1^t (1 + Kd_t)}$$

It is easy to show that:

[60] $D_1 - D_0 = N_1 - N_0 + D_0\, Kd_1 - N_0\, r_1$

Consequently: $\Delta D = \Delta N + D_0\, Kd_1 - N_0 r_1$

Taking into account this expression and equations [51] and [52], we obtain:

[61] $CCF_t = FCF_t + N_{t-1}\, r_t\, T$

The expression for WACC and $WACC_{BT}$ in this case is:

[62] $$WACC = \frac{E\, Ke + D\, Kd - N\, r\, T}{E + D} \qquad WACC_{BT} = \frac{E\, Ke + D\, Kd}{E + D}$$

The expression for VTS in this case is:

[63]
$$VTS_0 = \sum_{t=1}^{\infty} \frac{D_{t-1}\,Ku_t\,T - (N_{t-1}\,r_t - D_{t-1}\,Kd_t)T}{\prod_{1}^{t}(1+Ku_t)}$$

16. Impact on the valuation when D ≠ N, without cost of leverage

Table 10 shows the impact on the valuation of Font, Inc. if it is assumed that D is not equal to N. In order to calculate the debt's market value (D), the following expressions are used in Table 10:

$$\text{Debt} = \sum_{i=1}^{10} \frac{\text{Cash flow to debt}_i}{\prod_{j=1}^{j=i}(1+Kd_j)} + \frac{\text{Cash flow to debt}_{11}}{(Kd - g)} \times \frac{1}{\prod_{j=1}^{10}(1+Kd_j)} \qquad \beta d_i = \frac{Kd_i - Rf_i}{Rm_i - Rf_i}$$

Table 10. Valuation of Font, Inc. assuming that D ≠ N
$Kd = R_F + (Ku - R_F) \times D(1-T) / [D(1-T)+E)$

		0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
35	**Ku**	**20%**	**20%**	**20%**	**20%**	**20%**	**20%**	**20%**	**20%**	**20%**	**20%**	**20%**
38	Vu without taxes (Ku)	2,91	3,080.6	3,826.7	4,172.0	4,336.4	4,483.7	4,800.4	5,034.5	5,280.6	5,543.4	5,820.5
9	N	1,800	1,800	2,300	2,300	2,050	1,800	1,700	1,450	1,200	1,000	1,050
39	**r**	***15%***	***15%***	***15%***	***15%***	***15%***	***15%***	***15%***	***15%***	***15%***	***15%***	***15%***
A	**D**	**1,704.4**	**1,729.1**	**2,255.4**	**2,299.8**	**2,093.9**	**1,879.2**	**1,805.3**	**1,576.5**	**1,340.5**	**1,149.8**	**1,207.3**
40	**Kd**	29%	14%	26%	16.92%	16.37%	15.76%	15.30%	14.68%	14.12%	13.70%	13.70%
B	Beta d	0.6609	0.6425	0.6577	0.6152	0.5464	0.4696	0.4123	0.3354	0.2653	0.2122	0.2122
C	Nr-DKd		-24.6432	-26.3667	-44.3261	-44.1592	-35.3068	-26.0991	-21.1851	-13.9785	-9.3106	-7.4897
D	Ke - Kd	8%	8%	8%	8%	8%	8%	8%	8%	8%	8%	8%
E	D T Ku + (Nr-DKd)*T		110.68	111.81	142.37	145.53	134.22	122.41	118.96	105.46	90.58	77.86
41	VTS	593.27	601.24	609.68	589.25	561.57	539.67	525.19	511.27	508.06	519.09	545.05
42	VTS + Vu	2,272.91	2,354.31	3,018.37	3,234.68	3,223.59	3,259.09	3,477.99	3,607.23	3,753.20	3,925.24	4,121.50
43	**-D=E1**	**568**	**625**	**763**	**935**	**1,130**	**1,380**	**1,673**	**2,031**	**2,413**	**2,775**	**2,914**
44	Beta E	1.6609	1.6425	1.6577	1.6152	1.5464	1.4696	1.4123	1.3354	1.2653	1.2122	1.2122
45	**Ke**	**25.29%**	**25.14%**	**25.26%**	**24.92%**	**24.37%**	**23.76%**	**23.30%**	**22.68%**	**22.12%**	**21.70%**	**21.70%**
46	**E 2 = PV(Ke;ECF)**	**568**	**625**	**763**	**935**	**1,130**	**1,380**	**1,673**	**2,031**	**2,413**	**2,775**	**2,914**
47	$E_t = E_{t-1}$ (1+Ke) - ECF	568	625	763	935	1,130	1,380	1,673	2,031	2,413	2,775	2,914
48	**Reformed WACC**	**15.13%**	**15.25%**	**15.28%**	**15.50%**	**15.84%**	**16.24%**	**16.58%**	**08%**	**59%**	**18.02%**	**18.02%**
49	PV(WACC;FCF)	2,272.91	2,354.31	3,018.37	3,234.68	3,223.59	3,259.09	3,477.99	3,607.23	3,753.20	3,925.24	4,121.50
50	**- D = E 3**	**568**	**625**	**763**	**935**	**1,130**	**1,380**	**1,673**	**2,031**	**2,413**	**2,775**	**2,914**
51	**$WACC_{BT}$**	**19.29%**	**19.26%**	**19.28%**	**19.23%**	**19.18%**	**19.14%**	**19.15%**	**19.19%**	**19.27%**	**19.35%**	**19.35%**
52	PV($WACC_{BT}$;CCF)	2,272.91	2,354.31	3,018.37	3,234.68	3,223.59	3,259.09	3,477.99	3,607.23	3,753.20	3,925.24	4,121.50
53	**- D = E 4**	**568**	**625**	**763**	**935**	**1,130**	**1,380**	**1,673**	**2,031**	**2,413**	**2,775**	**2,914**

The most significant differences between Tables 8 and 10 are:

(million euros)	**Table 8**	**Table 10**
Value of debt D	**1,800**	**1,705**
Value of equity E	**506**	**568**
Value of State's interest	**611**	**644**
TOTAL	**2,917**	**2,917**

Impact on the valuation when D ≠ N, with cost of leverage, in a real-life case

The simplified formulae for the levered beta are: [27] and [28]. If these simplified formulae are used, the levered beta (β_L*) will be greater than that obtained using the full formula [17].

In addition, the value of the equity (E* or E') will be less than that obtained previously (E) because the required return to equity now (Ke* or Ke') is greater than that used previously (Ke). Logically, the weighted cost of debt and equity now (WACC') is greater than that used previously (WACC).

With these simplifications, we introduce cost of leverage in the valuation: in formula [50], we must consider the term "Cost of Leverage", which represents the cost of bankruptcy (increased probability of bankruptcy) and/or a decrease of the expected FCF when the debt ratio is increased.

We assume that the debt's market value is the same as its nominal value. The most important differences in the valuation are shown in Table 11 and Figures 2 and 3.

The value of the equity is 506 million with the full formula, 332 million with the abbreviated formula [28] and 81 million with the abbreviated formula [27].

Note that, in parallel with formulae [29] and [30]:

$$506 - 332 = 174 = \sum_{t=1}^{\infty} \frac{D_{t-1}(1-T)(Kd_t - R_F)}{\prod_1^t (1+Ku_t)}$$

$$506 - 81 = 425 = \sum_{t=1}^{\infty} \frac{D_{t-1}[T(Ku - R_F) + (1-T)(Kd - R_F)]}{\prod_1^t (1+Ku_t)}$$

Where:

$$332 = \sum_{t=1}^{\infty} \frac{ECF'_t}{\prod_1^t (1+Ke_t)} = \sum_{t=1}^{\infty} \frac{ECF_t}{\prod_1^t (1+Ke'_t)} \qquad 81 = \sum_{t=1}^{\infty} \frac{ECF_t^*}{\prod_1^t (1+Ke_t)} = \sum_{t=1}^{\infty} \frac{ECF_t}{\prod_1^t (1+Ke_t^*)}$$

Table 11. Impact of the use of the simplified formulae on the valuation of Font Inc.

Year	0	1	2	3	4	5	6	7	8	9	10
ECF = Div.		87.00	19.50	20.75	38.25	25.13	35.00	31.65	78.65	171.02	463.42
FCF		262.50	-305.00	245.00	512.50	475.00	310.50	447.40	470.02	488.02	510.92
N	1800	1800	2300	2300	2050	1800	1700	1450	1200	1000	1050
r	15%	15%	15%	15%	15%	15%	15%	15%	15%	15%	15%
E	506	579	734	935	1,158	1,431	1,741	2,113	2,504	2,873	3,016
E´	332	405	560	771	1,006	1,289	1,605	1,983	2,376	2,743	2,880
E*	81	154	310	535	788	1,084	1,410	1,796	2,193	2,556	2,684
Beta E	2.44	2.26	2.27	2.00	1.72	1.51	1.40	1.28	1.19	1.14	1.14
Beta E´	4.53	3.89	3.67	2.94	2.32	1.91	1.69	1.48	1.33	1.24	1.24
Beta E*	23.20	12.66	8.43	5.30	3.60	2.66	2.21	1.81	1.55	1.39	1.39
Ke	31.6%	30.1%	30.2%	28.0%	25.8%	24.1%	23.2%	22.2%	21.6%	21.1%	21.1%
Ke´	48.2%	43.1%	41.4%	35.5%	30.6%	27.3%	25.5%	23.8%	22.6%	21.9%	21.9%
Ke*	197.6%	113.3%	79.4%	54.4%	40.8%	33.3%	29.7%	26.5%	24.4%	23.1%	23.1%
ECF		87.00	19.50	20.80	38.30	25.10	35.00	31.60	78.60	171.00	463.40
ECF´		51.90	-15.60	-24.10	-6.60	-14.90	-0.10	-1.50	50.40	147.60	443.90
ECF*		1.50	-66.00	-88.50	-71.00	-72.30	-50.50	-49.10	9.80	114.00	415.90
Ku	20%	20%	20%	20%	20%	20%	20%	20%	20%	20%	20%
Ku´	22.34%	22.23%	22.18%	21.98%	21.71%	21.43%	21.22%	20.97%	20.74%	20.57%	20.57%
Ku*	26.83%	26.46%	26.05%	25.38%	24.59%	23.79%	23.21%	22.51%	21.92%	21.48%	21.48%
ßu	1	1	1	1	1	1	1	1	1	1	1
ßu´	1.29	1.28	1.27	1.25	1.21	1.18	1.15	1.12	1.09	1.07	1.07
ßu*	1.85	1.81	1.76	1.67	1.57	1.47	1.4	1.31	1.24	1.19	1.19
WACC	14.54%	14.70%	14.69%	15.02%	15.53%	16.10%	16.54%	15%	73%	18.19%	18.19%
WACC´	15.74%	15.88%	15.94%	16.22%	16.61%	06%	40%	87%	18.31%	18.65%	18.65%
WACC*	85%	93%	18.02%	18.18%	18.37%	18.60%	18.77%	19.00%	19.20%	19.37%	19.37%

Figure 2. Impact of the use of the simplified formulae on the required return to equity of Font, Inc.

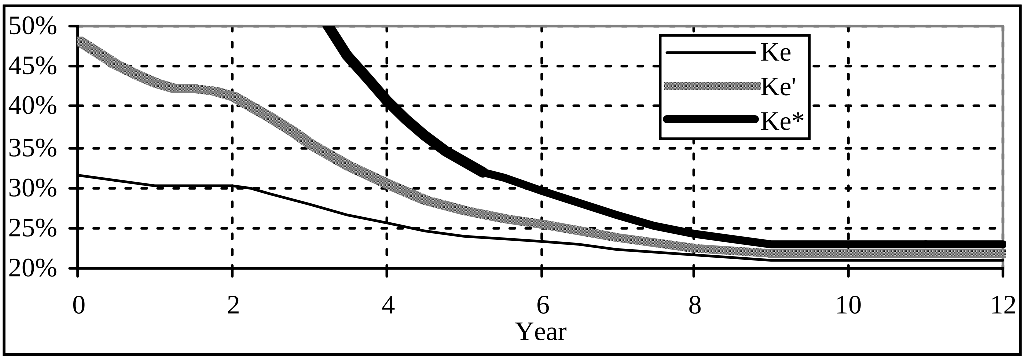
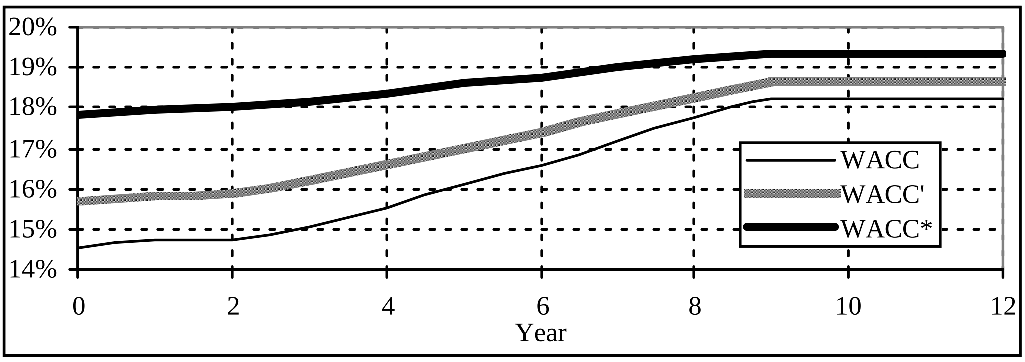


Figure 3. Impact of the use of the simplified formulae on the WACC of Font, Inc.



Appendix 1. Main valuation formulae

Valuation formulae

	Perpetuities (g=0)	Constant growth	General case
E	$E = \frac{ECF}{Ke}$	$E = \frac{ECF_1}{Ke - g}$	E = PV [ECF; Ke]
D	$D = \frac{I}{Kd}$	$D_0 = \frac{(I - \Delta D)_1}{Kd - g} = \frac{KdD_0 - gD_0}{Kd - g}$	$D_0 = \sum_{t=1}^{\infty} \frac{D_{t-1} Kd_t - (D_t - D_{t-1})}{\prod_1^t (1 + Kd_t)}$
E+D	$E + D = \frac{FCF}{WACC}$	$E + D = \frac{FCF_1}{WACC - g}$	E + D = PV [FCF; WACC]
E+D	$E + D = \frac{CCF}{WACC_{BT}}$	$E + D = \frac{CCF_1}{WACC_{BT} - g}$	E + D = PV [CCF; $WACC_{BT}$]
APV	$E + D = \frac{FCF}{Ku} + DVTS - CL$	$E + D = \frac{FCF_1}{Ku - g} + DVTS - CL$	E + D = PV [FCF; Ku] + VTS - CL
if CL=0	VTS = DT	VTS = D Ku T / (Ku - g)	VTS = PV [D Ku T; Ku]
VTS if CL=0 $r \neq Kd$	VTS = DT	$VTS = \frac{D\,T\,Ku + T\,[Nr - D\,Kd]}{Ku - g}$	$VTS_0 = \sum_{t=1}^{\infty} \frac{D_{t-1} Ku_t T - (N_{t-1} r_t - D_{t-1} Kd_t) T}{\prod_1^t (1 + Ku_t)}$
if CL=0	$K_{TU} = Ku$ $K_{TL} = Ke$	$K_{TU} \neq Ku$ $K_{TL} \neq Ke$	$KTU \neq Ku$ $KTU \neq Ke$

Flows relationships

	Perpetuities (g=0)	Constant growth	General case
r = Kd	ECF = FCF - D Kd (1-T) CCF = ECF + D Kd CCF = FCF - D Kd T	$ECF_1 = FCF_1$ - Do [Kd (1 - T) - g] $CCF_1 = ECF_1$ + Do (Kd - g) $CCF_1 = FCF_1$ - Do Kd T	$ECF_t = FCF_t + \Delta D_t - I_t (1 - T)$ $CCF_t = ECF_t - \Delta D_t + I_t$ $CCF_t = FCF_t + I_t T$
$r \neq Kd$	D Kd = N r	D = N (r-g) / (Kd-g) ECF = FCF - Nr (1-T) + gN ECF = FCF - D (Kd - g) + N r T $CCF_t = FCF_t + N_{t-1} r_t T$	$D_0 = \sum_{t=1}^{\infty} \frac{N_{t-1} r_t - (N_t - N_{t-1})}{\prod_1^t (1 + Kd_t)}$

If D=N	$WACC = \frac{E\,Ke + D\,Kd\,(1 - T)}{E + D}$	$WACC_{BT} = \frac{E\,Ke + D\,Kd}{E + D}$
If D≠N	$WACC = \frac{E\,Ke + D\,Kd - N\,r\,T}{E + D}$	$WACC_{BT} = \frac{E\,Ke + D\,Kd}{E + D}$

	CL = 0	CL > 0 (β´)	CL >> 0 (β*)
$ß_L$	$\beta_L = \beta u + \frac{D(1 - T)}{E}(\beta u - \beta d)$	$\beta'_L = \beta u + \frac{D\,(1 - T)}{E'}\beta u$	$\beta *_L = \beta u + \frac{D}{E *}\beta u$

$VTS_t = GU_t - GL_t = E_t + D_t - Vu_t$	$Ku = R_F + \beta u\, P_M$	$Kd = R_F + \beta_d\, P_M$	$Ke = R_F + \beta_L\, P_M$

Appendix 2. A formula for the required return to debt

Formula [12] tells us the relationship that must exist between Ku, Ke and Kd for each level of debt (assuming that the probability of bankruptcy is zero), but we have not found any formula that tells us how to calculate Kd from the company's risk (Ku) and debt ratio. Kd can be interpreted as the "reasonable" return that bondholders or the bank must (or should) demand, considering the company's risk and the size of the debt. For the moment, we are assuming that Kd is also the interest paid by the company on its debt.

The case of maximum debt. When all the cash flow generated by the assets corresponds to debt (ECF = 0), in the absence of leverage costs[20], the debt's risk at this point must be identical to the assets' risk, that is, Kd = Ku.

The case of minimum debt. On the other hand, for a minimum debt, the cost must be R_F.

A description of the debt's cost that meets these two conditions is:

[64] $Kd = R_F + D(1 - T) (Ku- R_F)/[D(1 - T) + E]$

which implies

[65] $\beta_d = \beta_U \; D(1-T) / [D(1 - T) + E]$

Substituting [64] in [16] gives:

[66] $Ke = Ku + D(1 - T) (Ku- R_F)/[D(1 - T) + E] = Ku + Kd- R_F$

---

[20] This can only happen if the owners of the debt and the equity are the same.

References

Damodaran, Aswath (1994), *Damodaran on Valuation*, John Wiley and Sons, New York.

Fernández, Pablo (2004), "The Value of Tax Shields is NOT Equal to the Present Value of Tax Shields", *Journal of Financial Economics* (July), Vol. 73/1, pp. 145-165.

Fernández, Pablo (2005), "The Value of Tax Shields is not Equal to the Present Value of Tax Shields: A Correction", SSRN working paper No. 651206.

Harris, R.S. and J.J. Pringle (1985), "Risk-Adjusted Discount Rates Extensions Form the Average-Risk Case", *Journal of Financial Research* (Fall), pp. 237-244.

Kaplan, S. and R. Ruback (1995), "The Valuation of Cash Flow Forecast: An Empirical Analysis", *Journal of Finance*, Vol. 50, No 4, September.

Luehrman, Timothy A. (1997), "What's It Worth? A General Manager's Guide to Valuation", and "Using APV: A Better Tool for Valuing Operations", *Harvard Business Review*, (May-June), pp. 132-154.

Modigliani, F. and M. Miller (1958), "The Cost of Capital, Corporation Finance and the Theory of Investment", *American Economic Review*, 48, pp. 261-297.

Modigliani, F. and M. Miller (1963), "Corporate Income Taxes and the Cost of Capital: A Correction", *American Economic Review* (June), pp. 433-443.

Myers, S.C. (1974), "Interactions of Corporate Financing and Investment Decisions – Implications for Capital Budgeting", *Journal of Finance* (March), pp. 1-25

Ruback, Richard S. (1995), "A Note on Capital Cash Flow Valuation", Harvard Business School, 9-295-069.

Ruback, Richard S. (2002), "Capital Cash Flows: A Simple Approach to Valuing Risky Cash Flows", *Financial Management* (Summer), pp. 85-103.

Tham, Joseph, and Ignacio Vélez-Pareja (2001), "The correct discount rate for the tax shields: the N-period case", SSRN Working Paper.