



University of Navarra

CIIF

# *Working Paper*

---

WP No 576  
November, 2004

REPLY TO:  
*"THE VALUE OF TAX SHIELDS IS EQUAL TO  
THE PRESENT VALUE OF TAX SHIELDS"*

Pablo Fernández\*

---

\* Professor of Financial Management, PricewaterhouseCoopers Chair of Finance, IESE

IESE Business School – Universidad de Navarra

Avda. Pearson, 21 – 08034 Barcelona. Tel.: (+34) 93 253 42 00 Fax: (+34) 93 253 43 43

Camino del Cerro del Águila, 3 (Ctra. de Castilla, km. 5,180 – 28023 Madrid. Tel.: (+34) 91 357 08 09 Fax: (+34) 91 357 29 13

Copyright © 2004 IESE Business School

The CIIF, International Center for Financial Research, is an interdisciplinary center with an international outlook and a focus on teaching and research in finance. It was created at the beginning of 1992 to channel the financial research interests of a multidisciplinary group of professors at IESE Business School and has established itself as a nucleus of study within the School's activities.

Ten years on, our chief objectives remain the same:

- Find answers to the questions that confront the owners and managers of finance companies and the financial directors of all kinds of companies in the performance of their duties
- Develop new tools for financial management
- Study in depth the changes that occur in the market and their effects on the financial dimension of business activity

All of these activities are programmed and carried out with the support of our sponsoring companies. Apart from providing vital financial assistance, our sponsors also help to define the Center's research projects, ensuring their practical relevance.

The companies in question, to which we reiterate our thanks, are:

Aena, A.T. Kearney, Caja Madrid, Fundación Ramón Areces, Grupo Endesa, Telefónica and Unión Fenosa.

<http://www.iese.edu/ciif/>

**REPLY TO**  
***“THE VALUE OF TAX SHIELDS IS EQUAL TO***  
***THE PRESENT VALUE OF TAX SHIELDS”***

**Abstract**

In a recent paper, Cooper and Nyborg (2004) argue that the results of Fernández (2004) are wrong because value-additivity is violated and because “Fernández paper comes from mixing the Miles-Ezzell leverage policy with the Miller-Modigliani leverage adjustment.”

Cooper and Nyborg’s paper is thought-provoking and helps a lot in rethinking the value of tax shields. However, their conclusions are not correct because, as will be proven below, the main result of Fernández (2004) is correct for several situations.

An evident error of Cooper and Nyborg (2004) is that their formulae (4), (6), (8) and (11), which they attribute to Miles and Ezzell (1980), correspond to Harris and Pringle (1985) and Ruback (2002). In addition, their formulae (3) and (5) are not general: they are valid only for perpetuities without growth.

In this paper I also show that the value of tax shields depends only upon the nature of the stochastic process of the net increase of debt.

Classification JEL: G12, G31, G32

**Keywords:** Value of tax shields, Present value of the net increases of debt, unlevered beta, levered beta, leverage cost.

**REPLY TO**  
***“THE VALUE OF TAX SHIELDS IS EQUAL TO***  
***THE PRESENT VALUE OF TAX SHIELDS”***

The first evident error of Cooper and Nyborg (2004) is that, throughout the paper, they mention the Miles-Ezzell theory, and yet their formulae (4), (6), (8) and (11), which they attribute to Miles-Ezzell (1980), in fact correspond to Harris and Pringle (1985) and Ruback (2002).

The second error of Cooper and Nyborg (2004) is that their formulae (3) and (5) are not general: they are valid only for perpetuities without growth.

The third error of Cooper and Nyborg (2004) is to maintain that the main conclusion of Fernández (2004)<sup>1</sup> violates value-additivity. Value-additivity is not violated because the value of tax shields is the difference between the present values of two different cash flows, each with its own risk: the present value of taxes for the unlevered company and the present value of taxes for the levered company<sup>2</sup>.

There is also a subtle difference between the Miles-Ezzell (1980) assumption about the capital structure, namely,  $D = K E$ , and the assumption that I use in my paper:  $E\{D\} = K E\{E\}$ , where  $E\{\cdot\}$  is the expected value operator,  $D$  is the value of debt, and  $E$  is the equity value, and  $K$  is a constant. The Miles-Ezzell (1980) assumption requires continuous debt rebalancing, while my assumption does not.

### **1. Value of tax shields and the stochastic process of net debt increases**

For simplicity, Fernández (2004) neglected to use expected value notation. The equations of Fernández (2004) that are affected by using the expected value notation, where  $E\{\cdot\}$  is the expected value operator, are:

$$ECF_t = PAT_{L_t} - \Delta NFA_t - \Delta WCR_t + \Delta D_t \quad (5a)$$

---

<sup>1</sup> The value of tax shields is NOT equal to the present value of tax shields: the value of tax shields is the difference between the present values of two different cash flows, each with its own risk: the present value of taxes for the unlevered company and the present value of taxes for the levered company.

<sup>2</sup> A similar situation is the valuation of a security that has promised cash flows proportional to the Microsoft and GE free cash flows, being long in Microsoft and short in GE. It is obvious that the value of that security is the difference between the present values of two different cash flows, each with its own risk: the present value of Microsoft free cash flow and the present value of GE free cash flow. It makes no sense to value this security as the present value of the expected difference between the cash flows.

Where  $\Delta WCR_t = WCR_t - WCR_{t-1}$  = Increase of Working Capital Requirements in period t.  
 $\Delta NFA_t = NFA_t - NFA_{t-1}$  = Increase of Net Fixed Assets in period t.  
 $\Delta D_t = D_t - D_{t-1}$  = Increase of Debt in period t.

$$FCF_t = PATu_t - \Delta NFA_t - \Delta WCR_t \quad (7a)$$

$$\text{Taxes}u_t = [T/(1+T)] PATu = [T/(1+T)] (FCF_t + \Delta NFA_t + \Delta WCR_t) \quad (9a)$$

$$\text{Taxes}L_t = [T/(1+T)] (ECF_t + \Delta NFA_t + \Delta WCR_t - \Delta D_t) \quad (12a)$$

Below, the convention is used of referring to the equation numbers in Fernández (2004). For no growth perpetuities,  $E\{\Delta NFA_t\} = E\{\Delta WCR_t\} = E\{\Delta D_t\} = 0$ , and equations (5), (7), (9) and (12) in Fernández (2004) are equal to equations (5a), (7a), (9a) and (12a) above.

For growing perpetuities,

$E\{\Delta NFA_1\} + E\{\Delta WCR_1\} - E\{\Delta D_1\} = g(NFA + WCR - D) = gEb_v$ , and  
 $E\{\Delta NFA_1\} + E\{\Delta WCR_1\} = g(NFA + WCR) = g(Eb_v + D)$ , which makes equations (24) and (22) in Fernández (2004) correct.

Define  $PV_0[\cdot]$  as the present value operator. The present values at  $t=0$  of equations (9) and (12) are:

$$Gu_0 = [T/(1+T)] (Vu_0 + PV_0[\Delta NFA_t + \Delta WCR_t]) \quad (11a)$$

$$GL_0 = [T/(1+T)] (E_0 + PV_0[\Delta NFA_t + \Delta WCR_t] - PV_0[\Delta D_t]) \quad (14a)$$

(11a) is equal to (11) only if  $PV_0[\Delta NFA_t + \Delta WCR_t] = 0$ . In this situation, equation (10) holds. Analogously, (14a) is equal to (14) only if  $PV_0[\Delta NFA_t + \Delta WCR_t - \Delta D_t] = 0$  and equation (13) holds. But there are situations in which, for no growth perpetuities,  $PV_0[\Delta NFA_t + \Delta WCR_t] < 0$ .

The value of tax shields comes from the difference between (11a) and (14a):

$$VTS_0 = Gu_0 - GL_0 = [T/(1+T)] (Vu_0 - E_0 + PV_0[\Delta D_t])$$

As, according to equation (1),  $Vu_0 - E_0 = D_0 - VTS_0$ ,

$VTS_0 = [T/(1+T)] (D_0 - VTS_0 + PV_0[\Delta D_t])$ . And the value of tax shields is:

$$VTS_0 = T \cdot D_0 + T \cdot PV_0[\Delta D_t] \quad (16a)$$

Equation (16a) shows that the value of tax shields depends only upon the nature of the stochastic process of the net increase of debt<sup>3</sup>. The problem of equation (16a) is how to calculate  $PV_0[\Delta D_t]$ , which requires knowing the appropriate discount rate to apply to the increase of debt.

---

<sup>3</sup> If the nominal value of debt (N) is not equal to the value of debt (D), because the interest rate (r) is different from the required return to debt flows (Kd), equation (16a) is:  $VTS_0 = T \cdot D_0 + T \cdot PV_0[\Delta N_t]$ .

The relationship between D and N is:  $D_0 = PV_0[\Delta N_t] + PV_0[N_t \cdot r_t]$ .

If a company has scant access to banks or financial markets, these difficulties may be solved by paying a high cost of debt. In these situations,  $D > N$ .

## 2. Value of tax shields in specific situations

It is illustrative to apply (16a) to specific situations.

### 2.1. Perpetual debt

If the debt is a constant perpetuity (a consol),  $PV_0[\Delta D_t] = 0$ , and

$$VTS_0 = T \cdot D_0 \quad (16)$$

This result is far from being a new idea. Brealey and Myers (2000), Modigliani and Miller (1963), Copeland et al. (2000), Fernández (2004) and many others report it. However, the way of reaching this result is new.

### 2.2. Debt of one-year maturity but perpetually rolled-over

As in the previous case,  $E\{D_t\} = D_0$ , but the debt is expected to be rolled-over each year. The appropriate discount rate for the cash flows due to the existing debt is  $K_d$ .<sup>4</sup> Define  $K_{ND}$  as the appropriate discount rate for the new debt that must be obtained each year, then:

Present value of obtaining the new debt each year<sup>5</sup> =  $D_0 / K_{ND}$

Present value of the principal repayments at the end of each year<sup>6</sup> =  $D_0 (1 + K_{ND}) / [(1+K_d) K_{ND}]$

$PV_0[\Delta D_t]$  is the difference of these two expressions. Then:

$$PV_0[\Delta D_t] = - D_0 (K_{ND} - K_d) / [(1+K_d) K_{ND}] \quad (50)$$

If  $K_{ND} = K_d$ , then  $PV_0[\Delta D_t] = 0$

In a constant perpetuity ( $E\{FCF_t\} = FCF_0$ ), it seems reasonable that, if we do not expect credit rationing,  $K_{ND} = K_d$ , which means that the risk associated with the repayment of the current debt and interest ( $K_d$ ) is equivalent to the risk associated with obtaining an equivalent amount of debt at the same time ( $K_{ND}$ ).

### 2.3. Debt increases are as risky as the free cash flows

In this case, the correct discount rate for the expected increases of debt is  $K_u$ , the required return to the unlevered company. In the case of a constant growing perpetuity,

$$PV_0[\Delta D_t] = g \cdot D_0 / (K_u - g),$$

and the VTS is:

<sup>4</sup> We use  $K_d$  so as not to complicate the notation. It should be  $K_{d_t}$ , a different rate following the yield curve. Using  $K_d$  we may also think of a flat yield curve.

<sup>5</sup> Present value of obtaining the new debt each year =  $D / (1+K_{ND}) + D / (1+K_{ND})^2 + D / (1+K_{ND})^3 + \dots$  because  $D = E\{D_t\}$ , where  $D_t$  is the new debt obtained at the end of year  $t$  (beginning of  $t+1$ ).

<sup>6</sup> The present value of the principal repayment at the end of year 1 is  $D / (1+K_d)$ .  
The present value of the principal repayment at the end of year 2 is  $D / [(1+K_d)(1+K_{ND})]$ .  
The present value of the principal repayment at the end of year  $t$  is  $D / [(1+K_d)(1+K_{ND})^{t-1}]$ .  
Because  $D = E\{D_t\}$ , where  $D_t$  is the debt repayment at the end of year  $t$ .

$$VTS_0 = T \cdot Ku \cdot D_0 / (Ku - g) \quad (28)$$

For  $g = 0$ , equations (28) and (16) are equal.

Equation (28) is the main one in Fernández (2004), although the way of deriving it is different.

#### ***2.4. The company is expected to repay the current debt without issuing new debt***

In this situation, the appropriate discount rate for the negative  $\Delta D_t$  (principal payments) is  $K_d$ , the required return to the debt. In this situation, Myers (1974) applies:

$$PV_0[\Delta D_t] = PV_0[E\{\Delta D_t\}; K_d],$$

and the VTS is:

$$VTS_0 = D_0 \cdot T + T \cdot PV_0[E\{\Delta D_t\}; K_d] \quad (51)$$

For perpetual debt, equations (51), (28) and (16) are equal.

For a company that is expected to repay the current debt without issuing new debt, the value of the debt today is:  $D_0 = PV_0[E\{D_{t-1}\} \cdot K_d - E\{\Delta D_t\}; K_d]$ .

Substituting this expression in (51), we get the Myers (1974) formula:

$$VTS_0 = PV_0[T \cdot E\{D_{t-1}\} \cdot K_d; K_d]$$

#### ***2.5. Debt is proportional to the Equity value***

This is the assumption made by Miles and Ezzell (1980), who claim that if  $D_t = L \cdot E_t$ , then the value of tax shields for perpetuities growing at a constant rate  $g$  is:

$$VTS_0 = \frac{D_0 K_d T (1 + Ku)}{(Ku - g) (1 + Kd)} \quad (100)$$

Substituting (100) in (16a), we get:

$$PV_0[\Delta D_t] = D_0 \frac{(Kd - Ku) + g(1 + Kd)}{(Ku - g)(1 + Kd)} \quad (101)$$

For the no growth case ( $g = 0$ ), equation (101) is:

$$PV_0[\Delta D_t] = D (Kd - Ku) / [Ku(1 + Kd)] < 0.$$

Comparing this expression with equation (50), it is clear that Miles and Ezzell imply that  $K_{ND} = Ku$ .

(101) is zero for  $g = (Ku - Kd) / (1 + Kd)$  and negative for smaller growth rates. There is not much economic sense in this expression.

Furthermore, to assume  $D_t = L \cdot E_t$  is not a good description of the debt policy of a company because:

1. If the company pays a dividend  $Div_t$ , simultaneously the company should reduce debt in an amount  $\Delta D_t = -L \cdot Div_t$
2. If the equity value increases, then the company should increase its debt, while if the equity value decreases, then the company should reduce its debt. If the equity value is such that  $L \cdot E_t > \text{Assets of the company}$ , then the company should hold excess cash only for the sake of complying with the debt policy.

### 3. Value of net debt increases implied by the alternative theories

There is a considerable body of literature on the discounted cash flow valuation of firms. This section addresses the most salient papers, concentrating particularly on those papers that propose alternative expressions for the value of tax shields (VTS). The main difference between all of these papers and the approach proposed above is that most previous papers calculate the value of tax shields as the present value of the tax savings due to the payment of interest. Instead, the correct measure of the value of tax shields is the difference between two present values: the present value of taxes paid by the unlevered firm and the present value of taxes paid by the levered firm. We will show how these proposed methods result in inconsistent valuations of the tax shields.

Modigliani and Miller (1958, 1963) study the effect of leverage on firm value. Their famous Proposition 1 states that, in the absence of taxes, the firm's value is independent of its debt, i.e.,  $E + D = V_u$ , if  $T = 0$ . In the presence of taxes and for the case of a perpetuity, but with zero risk of bankruptcy, they calculate the value of tax shields by discounting the present value of the tax savings due to interest payments on risk-free debt at the risk-free rate ( $R_F$ ), i.e.,  $VTS = PV[E\{D \cdot T \cdot R_F\}; R_F] = D \cdot T$ . As indicated above, this result is the same as our Eq. (16) for the case of perpetuities, but it is neither correct nor applicable for growing perpetuities. Modigliani and Miller explicitly ignore the issue of the riskiness of the cash flows by assuming that the probability of bankruptcy was always zero.

Myers (1974) introduces the APV (adjusted present value) method, in which the value of the levered firm is equal to the value of the firm with no debt plus the present value of the tax savings due to the payment of interest. Myers proposes calculating the VTS by discounting the expected tax savings ( $D \cdot K_d \cdot T$ ) at the cost of debt ( $K_d$ ). The argument is that the risk of the tax savings arising from the use of debt is the same as the risk of the debt. The value of tax shields is  $VTS = PV[E\{D \cdot K_d \cdot T\}; K_d]$ . In section 2.4 we have shown that this expression is correct only when the company is expected to repay the current debt without issuing new debt.

Harris and Pringle (1985) propose that the present value of the tax savings due to the payment of interest should be calculated by discounting the expected interest tax savings ( $D \cdot K_d \cdot T$ ) at the required return to unlevered equity ( $K_u$ ), i.e.,  $VTS = PV[E\{D \cdot K_d \cdot T\}; K_u]$ . Their argument is that the interest tax shields have the same systematic risk as the firm's underlying cash flows and, therefore, should be discounted at the required return to assets ( $K_u$ ). Furthermore, Harris and Pringle believe that "the MM position is considered too extreme by some because it implies that interest tax shields are no more risky than the interest payments themselves" (p. 242). Ruback (1995, 2002) and Brealey and Myers (2000, p. 555) also claim that the appropriate discount rate for tax shields is  $K_u$ , the required return to unlevered equity.



Ruback (2002) presents the Capital Cash Flow (CCF) method and claims that the appropriate discount rate is  $K_u$ . The capital cash flow is equal to the free cash flow plus the interest tax shield ( $D \cdot K_d \cdot T$ ). According to Ruback (2002), the value of the debt today ( $D$ ) plus that of the shareholders' equity ( $E$ ) is equal to the expected capital cash flow (CCF) discounted at the weighted average cost of capital before tax ( $WACC_{BT}$ ):

$$E + D = PV[E\{CCF\}; WACC_{BT}].$$

The definition of  $WACC_{BT}$  is

$$WACC_{BT} = (E K_e + D K_d) / (E + D)$$

But Ruback (1995, 2002) assumes that  $WACC_{BT} = K_u$ . With this assumption, Ruback gets the same valuation as Harris and Pringle (1985) because

$$E + D = PV[E\{FCF\}; K_u] + PV[E\{D \cdot K_d \cdot T\}; K_u] = V_u + PV[E\{D \cdot K_d \cdot T\}; K_u]$$

Note that  $PV[E\{D \cdot K_d \cdot T\}; K_u]$  is the VTS according to Harris and Pringle (1985).

A large part of the literature argues that the value of tax shields should be calculated differently depending on the debt strategy of the firm. A firm that wishes to keep a constant D/E ratio must be valued differently from a firm that has a preset level of debt. Miles and Ezzell (1980) indicate that for a firm with a fixed debt target (i.e., a constant  $[D/(D+E)]$  ratio), the correct rate for discounting the tax savings due to debt is  $K_d$  for the first year and  $K_u$  for the tax savings in later years. Although Miles and Ezzell do not mention what the value of tax shields should be, this can be inferred from their equation relating the required return to equity with the required return for the unlevered company in Eq. (22) in their paper. This relation implies that  $VTS = PV[E\{D \cdot T \cdot K_d\}; K_u] (1 + K_u)/(1 + K_d)$ . Inselbag and Kaufold (1997) and Ruback (2002) argue that if the firm targets the dollar values of debt outstanding, the VTS is given by the Myers (1974) formula. However, if the firm targets a constant debt/value ratio, the value of the tax shields should be calculated according to Miles and Ezzell (1980). Finally, Taggart (1991) proposes to use Miles and Ezzell (1980) if the company adjusts to its target debt ratio once a year, and Harris and Pringle (1985) if the company adjusts to its target debt ratio continuously.

Damodaran (1994, p. 31) argues that if all the business risk is borne by the equity, then the formula relating the levered beta ( $\beta_L$ ) to the asset beta ( $\beta_u$ ) is  $\beta_L = \beta_u + (D/E) \beta_u (1 - T)$ . This formula is exactly the formula in Eq. (25), assuming that  $\beta_d = 0$ . One interpretation of this assumption is (Damodaran, 1994, p. 31) that "all of the firm's risk is borne by the stockholders (i.e., the beta of the debt is zero)." In some cases, it may be reasonable to assume that the debt has a zero beta. But then, as assumed by Modigliani and Miller (1963), the required return to debt should be the risk-free rate. This relation for the levered beta appears in many finance books and is widely used by many consultants and investment bankers as an attempt to include some leverage cost in the valuation: for a given risk of the assets ( $\beta_u$ ), this formula results in a higher  $\beta_L$  (and consequently a higher  $K_e$  and a lower equity value) than Eq. (25). In general, it is hard to accept that the debt has no risk and that the return on the debt is uncorrelated with the firm's return on assets. From Damodaran's expression for  $\beta_L$  it is easy to deduce the relation between the required return to equity and the required return to assets, i.e.,  $K_e = K_u + (D/E) (1 - T) (K_u - R_F)$ . Although Damodaran does not mention what the value of tax shields should be, his formula relating the levered beta to the asset beta implies that the value of tax shields is:

$$VTS = PV[K_u; D \cdot T \cdot K_u - D (K_d - R_F) (1 - T)].$$

Given the large number of alternative methods existing in the literature to calculate the value of tax shields, Copeland, Koller, and Murrin (2000, p. 482) assert that "the finance literature does not provide a clear answer about which discount rate for the tax benefit of

interest is theoretically correct.” They further conclude, “We leave it to the reader’s judgment to decide which approach best fits his or her situation.”

We propose three ways to compare and differentiate among the different approaches. One way is to calculate the value of tax shields for level perpetuities according to the different approaches. A second way is to check the implied present value of the net increases of debt in each of the different approaches. A third way is to check the implied relation between the unlevered and levered cost of equity in each of the different approaches. The levered cost of equity should always be higher than the cost of assets ( $K_u$ ), since equity cash flows are riskier than the free cash flows.

Table 1 summarizes the implications of these approaches for the value of tax shields in level perpetuities. It shows that only four out of the eight approaches compute the value of tax shield in perpetuities as  $DT$ . The other four approaches imply a lower value of tax shields than  $DT$ .

From equation (16a) the present value of the increases of debt is:

$$PV_0[\Delta D_t] = (VTS_0 - T \cdot D_0) / T$$

Applying this equation to the theories mentioned, we may construct the predictions that each of these theories have for  $PV_0[\Delta D_t]$ . These predictions are reported in Table 2.  $PV_0[\Delta D_t]$  for level perpetuities should be zero. That is the case only in Modigliani-Miller (1963), Myers (1974) and Fernández (2004).

As we have already argued, Myers (1974) should be used when the company will not issue new debt; Fernández (2004) when the company expects to issue new debt in the future; and Modigliani-Miller may be applied only if the debt is risk-free.

Table 3 summarizes the implications for the relation between the cost of assets and the cost of equity in growing perpetuities. Table 3 shows that not all of the approaches also satisfy the relation between the cost of equity and the cost of assets. The Modigliani and Miller (1963) and Myers (1974) approaches do not always give a higher cost of equity than the cost of assets. Myers obtains  $K_e$  lower than  $K_u$  if the value of the tax shields is higher than the value of debt. This happens when  $D \cdot T \cdot K_d / (K_d - g) > D$ , that is, when the growth rate is higher than the after-tax cost of debt:  $g > K_d (1 - T)$ . Please note also that in this situation, as the value of tax shields is higher than the value of debt, the equity ( $E$ ) is worth more than the unlevered equity ( $V_u$ ). This hardly makes any economic sense. Modigliani and Miller also provides the inconsistent result of  $K_e$  being lower than  $K_u$  if the value of the tax shields is higher than  $D [K_u - K_d (1 - T)] / (K_u - g)$ . This happens when either the leverage, the tax rate, the cost of debt, or the market risk premium are high.

#### 4. Conclusion

The two theories that make economic sense are Myers (1974) and Fernández (2004). As we have already argued, Myers (1974) should be used when the company will not issue new debt and Fernández (2004) when the company expects to issue new debt in the future. Both theories provide the same value for no-growth perpetuities.

This paper shows that the value of tax shields is:

$$VTS_0 = T \cdot D_0 + T \cdot PV_0[\Delta D_t]$$

The critical parameter for calculating the value of tax shields is the present value of the net increases of debt. It may vary for different companies, but in some special circumstances it may be calculated.

If the debt is a constant perpetuity (a consol),  $PV_0[\Delta D_t] = 0$ , and  $VTS_0 = T \cdot D_0$

If the company is expected to repay the current debt without issuing new debt, the appropriate discount rate for the negative  $\Delta D_t$  (because they are principal payments) is  $K_d$ , the required return to the debt. In this situation, Myers (1974) applies:  $PV_0[\Delta D_t] = PV_0[E\{\Delta D_t\}; K_d]$ . And the VTS is  $VTS_0 = D \cdot T + T \cdot PV_0[E\{\Delta D_t\}; K_d]$ .

If  $\Delta D_t = K \cdot FCF_t$ , then the correct discount rate for the increases of debt is  $K_u$ , the required return to the unleveraged company:  $PV_0[\Delta D_t] = PV_0[E\{\Delta D_t\}; K_u]$ . In the case of a constant growing perpetuity,  $PV_0[\Delta D_t] = g \cdot D_0 / (K_u - g)$ . And the VTS is:  $VTS_0 = T \cdot K_u \cdot D_0 / (K_u - g)$ .

The paper also shows that discounting the expected tax shields at the required return to unlevered equity, as suggested by Harris and Pringle (1985), Miles and Ezzell (1980), and Ruback (2002), is inconsistent.

Cooper and Nyborg's (2004) paper is thought provoking and helps a lot in rethinking the value of tax shields. However, their conclusions are not correct because, as will be proven below, the main result of Fernández (2004) is correct for several situations. An evident error of Cooper and Nyborg (2004) is that their formulae (4), (6), (8) and (11), which they attribute to Miles and Ezzell (1980), correspond to Harris and Pringle (1985) and Ruback (2002). In addition, their formulae (3) and (5) are not general: they are valid only for perpetuities without growth. Cooper and Nyborg (2004) maintain that the main conclusion of Fernández (2004) violates value-additivity. Value-additivity is not violated because the value of tax shields is the difference between the present values of two different cash flows, each with its own risk: the present value of taxes for the unlevered company and the present value of taxes for the levered company.

**Table 1. Comparison of value of tax shields (VTS) in perpetuities**

Only three out of the seven approaches correctly compute the value of the tax shield in perpetuities as DT.

The other four theories imply a lower value of the tax shield than DT.

Theories	VTS	VTS in perpetuities
<b>Correct method</b>	<b><math>D \cdot T + T \cdot PV_0[\Delta D_t]</math></b>	<b>DT</b>
Damodaran (1994)	$PV[E\{D \cdot T \cdot K_u - D(K_d - R_F)(1-T)\}; K_u]$	< DT
Harris-Pringle (1985), Ruback (1995)	$PV[E\{D \cdot T \cdot K_d\}; K_u]$	< DT
Myers (1974)	$PV[E\{D \cdot T \cdot K_d\}; K_d]$	DT
Miles-Ezzell (1980)	$PV[E\{D \cdot T \cdot K_d\}; K_u] (1+K_u) / (1+K_d)$	< DT
Modigliani-Miller (1963)	$PV[E\{D \cdot T \cdot R_F\}; R_F]$	DT
Fernández (2004)	$PV[E\{D \cdot T \cdot K_u\}; K_u]$	DT

$K_u$  = unlevered cost of equity

$K_d$  = required return to debt

T = corporate tax rate

D = debt value

$R_F$  = risk-free rate

$PV[E\{D \cdot T \cdot K_u\}; K_u]$  = present value of the expected value of  $D \cdot T \cdot K_u$  discounted at the rate  $K_u$

**Table 2. Present value of the increases of debt implicit in the most popular formulae for calculating the value of tax shields. Constant growing perpetuities at a rate  $g$**

	<b>PV<sub>0</sub>[ΔD<sub>t</sub>] for constant growing perpetuities at a rate <math>g</math></b>	<b>PV<sub>0</sub>[ΔD<sub>t</sub>] if <math>g=0</math></b>
Damodaran (1994)	$\frac{g \cdot D_0}{K_u - g} - \frac{D_0 (K_d - R_F) (1 - T)}{K_u - g} \frac{1}{T}$	$-\frac{D_0 (K_d - R_F) (1 - T)}{K_u} \frac{1}{T}$
Harris and Pringle (1985), Ruback (1995)	$\frac{g \cdot D_0}{K_u - g} - \frac{D_0 (K_u - K_d)}{K_u - g}$	$-\frac{D_0 (K_u - K_d)}{K_u}$
Myers (1974)	$g \cdot D_0 / (K_d - g)$	0
Miles and Ezzell (1980)	$\frac{g \cdot D_0}{K_u - g} - \frac{D_0 (K_u - K_d)}{(K_u - g)(1 + K_d)}$	$-\frac{D_0 (K_u - K_d)}{K_u(1 + K_d)}$
Modigliani-Miller (1963)	$g \cdot D_0 / (R_F - g)$	0
Fernández (2004)	$g \cdot D_0 / (K_u - g)$	0

**Table 3. Comparison of the relation between Ke (levered cost of equity) and Ku (unlevered cost of equity) for growing perpetuities**

The approaches of Modigliani and Miller (1963) and Myers (1974) do not always result in a higher cost of equity (Ke) than cost of assets (Ku). Myers (1974) obtains Ke lower than Ku if the value of tax shields is higher than the value of debt. This happens when the growth rate (g) is higher than the after-tax cost of debt, i.e.,  $g > K_d (1 - T)$ . Modigliani and Miller (1963) also provide the inconsistent result of Ke being lower than Ku if the value of tax shields is higher than  $D [K_u - K_d (1 - T)] / (K_u - g)$ . This happens when leverage, the tax rate, the cost of debt, or the market risk premium are high.

Theories	Ke (levered cost of equity)	Ke < Ku
Damodaran (1994)	$K_e = K_u + (D/E) (1 - T) (K_u - R_F)$	No
Harris-Pringle (1985), Ruback (1995)	$K_e = K_u + (D/E) (K_u - K_d)$	No
Myers (1974)	$K_e = K_u + (D - VTS) (K_u - K_d) / E$	<b>Yes,</b> if $g > K_d (1 - T)$
Miles-Ezzell (1980)	$K_e = K_u + \frac{D}{E} (K_u - K_d) \left[ 1 - \frac{T K_d}{1 + K_d} \right]$	No
Modigliani-Miller (1963)	$K_e = K_u + \frac{D}{E} [K_u - K_d(1 - T) - (K_u - g) \frac{VTS}{D}] *$	<b>Yes,</b> if $VTS > D [K_u - K_d(1 - T)] / (K_u - g)$
Fernández (2004)	$K_e = K_u + (D/E) (1 - T) (K_u - K_d)$	No

\* Valid only for growing perpetuities

D = debt value

E = equity value

g = growth rate

Kd = required return to debt

R<sub>F</sub> = risk-free rate

T = corporate tax rate

VTS = value of tax shields

## References

- Brealey, R.A. and S.C. Myers (2000), *Principles of Corporate Finance*. Sixth edition. McGraw-Hill, New York.
- Cooper, Ian A. and Kjell G. Nyborg (2004), “The value of tax shields IS equal to the present value of tax shields”, SSRN paper no. 603821.
- Copeland, T.E., Koller, T. and J. Murrin (2000), *Valuation: Measuring and Managing the Value of Companies*. Third edition. Wiley, New York.
- Damodaran, A. (1994), *Damodaran on Valuation*. John Wiley and Sons, New York.
- Fernández, P. (2004), “The value of tax shields is NOT equal to the present value of tax shields”, *Journal of Financial Economics* 73/1 (July), 145-165.
- Harris, R.S. and J.J. Pringle (1985), “Risk-adjusted discount rates extensions form the average-risk case”, *Journal of Financial Research* 8, 237-244.
- Inselbag, I. and H. Kaufold (1997), “Two DCF approaches for valuing companies under alternative financing strategies and how to choose between them”, *Journal of Applied Corporate Finance* 10, 114-122.
- Miles, J.A. and J.R. Ezzell, (1980), “The weighted average cost of capital, perfect capital markets and project life: a clarification”, *Journal of Financial and Quantitative Analysis* 15, 719-730.
- Modigliani, F. and M. Miller (1958), “The cost of capital, corporation finance and the theory of investment”, *American Economic Review* 48, 261-297.
- Modigliani, F. and M. Miller (1963), “Corporate income taxes and the cost of capital: a correction”, *American Economic Review* 53, 433-443.
- Myers, S.C. (1974), “Interactions of corporate financing and investment decisions – implications for capital budgeting”, *Journal of Finance* 29, 1-25.
- Ruback, R. (1995), “A note on capital cash flow valuation”, Harvard Business School Case No. 9-295-069.
- Ruback, R. (2002), “Capital cash flows: a simple approach to valuing risky cash flows”, *Financial Management* 31, 85-103.
- Taggart, R.A. Jr. (1991), “Consistent valuation and cost of capital. Expressions with corporate and personal taxes”, *Financial Management* 20, 8-20.