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OPTIMAL CAPITAL STRUCTURE: PROBLEMS WITH THE HARVARD AND DAMODARAN APPROACHES

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Abstract

In this paper we will present an analysis of the optimal capital structure using two examples: one proposed by the Harvard Business School and the other proposed by Damodaran.

First, we highlight certain inconsistencies in the debt and equity costs assumed by the Harvard Business School note from a number of viewpoints. We calculate the incremental cost of debt implied in Harvard's note and we find also inconsistencies: surprisingly, the last two debt increments have a cost of 14.75% and 18.5%, while the required return to equity in the unlevered company is 12%. With respect to the cost of debt, the inconsistency is not the cost of debt (the bank can charge whatever interest it likes) but in assuming that the debt's cost is the same as its required return (or that the debt's value equals its nominal value). We also calculate the required return to incremental equity cash flow implied in Harvard's note and we find that the required return first falls, then increases, and then falls again. The required incremental return should fall as the leverage decreases. The probability of bankruptcy almost doubles beyond the optimal capital structure. The difference between the required return to equity and the required return to debt decreases for debt levels above the optimal capital structure. It is also shown that assuming no leverage costs there is no optimal structure (the company's value increases with the debt ratio) and the difference between required return to equity and the required return to debt is constant.

Damodaran (1994) offers a similar approach to that of the Harvard Business School note, but applies it to a real company (Boeing in 1990) and assumes a constant cash flow growth. One problem with Damodaran's results is that the value of the firm (D+E) for debt ratios above 70% is less than the value of debt, which implies a negative value for equity. We calculate the incremental cost of debt implied in Damodaran's example. It can be seen that increasing debt to take the debt ratio from 30% to 40% implies contracting that debt at 21.5%, which is an enormous figure. Stranger still is the finding that the next debt increment (which has a higher risk) is cheaper: it costs 19%. An additional error in Damodaran's calculations is that he calculates the WACC using book values in the weighting, instead of market values. It is also shown that if it is assumed that the debt's market value is the same as its book value, then the capital structure that minimizes the WACC also maximizes the share price. However, without this assumption, the minimum value of the WACC may not occur at the same point as the maximum share price.

JEL Classification: G12, G31, M21

OPTIMAL CAPITAL STRUCTURE:
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Generally speaking, the optimal capital structure is considered to be that which minimizes the value of the weighted average cost of capital, WACC, and, consequently, maximizes the value of the firm, D+E[1]. We will see that if it is assumed that the debt's market value is the same as its book value, then the capital structure that minimizes the WACC also maximizes the share price. However, without this assumption, the minimum value of the WACC may not be the same as the maximum share price.

We will see that for an optimal structure to exist, it is necessary to assume that the firm's total value (debt + equity + present value of taxes) decreases with leverage. This may happen for two reasons: because the expected FCF decreases with debt level, or because the assets' risk (the FCF's risk and the likelihood of bankruptcy) increases with leverage[2] (or because of a combination of both).

In this paper we will present an analysis of the optimal structure using two examples: one proposed by the Harvard Business School and the other proposed by Damodaran.

1. Optimal structure according to a Harvard Business School technical note[3]

This note analyzes the relationships between the goal of maximizing each share's price and the objective of achieving an optimal capital structure, understanding this to be that which maximizes the firm's value (debt plus equity) and minimizes the weighted average cost of capital (WACC).

The note is based on Table 1, which illustrates a very simple example. A Company has invested 500,000 dollars in plant, machinery and working capital. The investment

1 It is meaningless to say that the optimal structure is that which maximizes the value of the firm (D+E). This value can be increased simply by asking the bank to increase the cost of debt because D+E = Vu + DVTS. Vu is constant and DVTS increases with higher interest payments.

2 This increase in the assets' risk may be due to their increased volatility or to the increased likelihood of bankruptcy.

3 This section discusses the technical note "Note on the Theory of Optimal Capital Structure", which was included in the book *Case Problems in Finance*, by Fruham et al. (1992). Irwin, 10th edition. This note is analyzed and criticized in the next section.

generates annual earnings before tax and interest (EBIT) amounting to 120,000 dollars to perpetuity. Annual depreciation is equal to new investments and the company distributes all its earnings as dividends. As the tax rate on profit is 50%, the free cash flow is 60,000 dollars to perpetuity.

Table 1. Optimal structure according to a Harvard Business School technical note

1	Book value debt ratio (leverage)	**0%**	**10%**	**20%**	**30%**	**40%**	**50%**
2	EBIT, earnings before interest and taxes	120,000	120,000	120,000	120,000	120,000	120,000
3	Interest	0	4,125	8,750	14,625	22,000	31,250
4	Profit before taxes (PBT)	120,000	115,875	111,250	105,375	98,000	88,750
5	Taxes (50%)	60,000	57,938	55,625	52,688	49,000	44,375
6	Profit after taxes (PAT)	60,000	57,938	55,625	52,688	49,000	44,375
7	Dividends = ECF	60,000	57,938	55,625	52,688	49,000	44,375
8	Interest + dividends (3)+(7)	60,000	62,063	64,375	67,313	71,000	75,625
9	**Cost of debt: Kd**	**8.00%**	**8.25%**	**8.75%**	**9.75%**	**11.00%**	**12.50%**
10	**Required return on equity: Ke**	**12.00%**	**12.50%**	**13.00%**	**13.50%**	**14.50%**	**16.00%**
11	Market value of debt D. (3)/(9)	0	50,000	100,000	150,000	200,000	250,000
12	Market value of equity E. (7)/(10)	500,000	463,500	427,885	390,278	337,931	277,344
13	Market value of the firm. (11)+(12)	500,000	513,500	527,885	540,278	537,931	527,344
14	Book value of debt, Dbv	0	50,000	100,000	150,000	200,000	250,000
15	Book value of equity, Ebv	500,000	450,000	400,000	350,000	300,000	250,000
16	Book value of the firm	500,000	500,000	500,000	500,000	500,000	500,000
17	Return on assets ROA = EBIT(1–T)/(16)	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%
18	Return on equity = (6)/(15)	12.00%	12.88%	13.91%	15.05%	16.33%	17.75%
19	Number of shares outstanding, NA	5,000	4,513	4,053	3,612	3,141	2,630
20	Price per share, P (12)/(19)	100	102,7	105,5769	**108,06**	107,5862	105,4688
21	Earnings per share, EPS. (6)/(19)	12	12.8375	13.725	14.5875	15.6	16.875
22	Price-earnings ratio, PER (20)/(21)	8.333333	8	7.692308	7.407407	6.896552	6.25
23	Book value debt ratio (14)/(16)	0%	10%	20%	30%	40%	50%
24	Market value debt ratio (11)/(13)	0.00%	9.74%	18.94%	27.76%	37.18%	47.41%
25	Weighted average cost of capital (WACC)	12.00%	11.68%	11.37%	**11.11%**	11.15%	11.38%
26	Free cash flow, FCF = EBIT (1-T)	60,000	60,000	60,000	60,000	60,000	60,000
27	Market value of the firm (26)/(25)	500,000	513,500	527,885	**540,278**	537,931	527,344

The company wants to select its capital structure from among the debt ratios shown in line 1 of Table 1.

Influence of leverage on payments to debt and equity. Lines 1-8 of Table 1 show the impact of the leverage on the company's income statement. In this example, the leverage does not influence the company's profit flows (EBIT) or its free cash flow (line 26). As debt is added to the capital structure, interest payments increase and profits (dividends) fall. Total payments to instrument holders (interest plus dividends) increase with the leverage. This increase arises from the discounted value of the tax shield.

Cost of funds. Lines 9 and 10 of Table 1 show the required return on debt and the required return on equity, that is, the return demanded by investors in order to purchase the company's debt and equity. As the leverage is increased, both debt and equity are exposed to a higher risk. The risk includes both the possibility of bankruptcy and a higher variability in the annual return. As the level of debt increases, investors demand a higher return in return for accepting the increased risk. The required return (lines 9 and 10) is the key assumption in the analysis of the optimal capital structure. The cost of debt is Kd (line 9), and the company's required return on equity is Ke (line 10). One important point to make is that the cost of debt may be information provided by banks or financial markets, but the required return on equity is an estimate.

Market value of debt and equity. In a perpetuity, the debt's market value (line 11) is equal to the annual interest payments, divided by the required return on debt (I/Kd). Likewise, equity's market value (line 12) is equal to the dividends divided by the required return to equity (Div/Ke). The market value of the company as a whole (line 13) is the sum of the market value of its debt and its equity. In the example, as debt is added to the capital structure, the company's market value (line 13) first increases and then decreases. The highest value for the company, 540,278 dollars, is attained with 150,000 dollars of debt.

Company return versus investor return. Lines 14 to 16 of Table 1 give the book value of debt and equity. It is assumed that the debt's book value is the same as its market value. Lines 17 and 18 show the company's ROA and ROE. The ROA is not affected by leverage and is always 12%. Without any debt, ROA = ROE, but when debt is added, the ROE moves above the ROA, according to the formula[4]:

$$ROE = ROA + [Dbv / Ebv]\,[ROA - Kd\,(1 - T)]$$

Dbv and Ebv represent the book value of debt and equity, respectively.

The ROE represents the return on equity's book value; however, the shareholders do not obtain this return, because their return depends on the market value. The shareholder return has very little bearing with the ROE.

Earnings per share and price-earnings ratios. Lines 19 and 20 show the number of shares outstanding and the price per share. The calculations are based on the assumption that, initially, the company has no debt and, in order to attain a certain level of leverage, the company issues debt and buys shares with the proceeds of the debt issue. The following sequence of events is assumed: 1) the company announces its intention to modify its long-term capital structure and issues debt; 2) its share price changes to reflect the company's new value, and 3) the company repurchases shares at the new price. The share price is obtained from the following equation: P = (E+D)/5,000[5].

Lines 21 and 22 of Table 1 show the earnings per share (EPS) and the PER. Logically, the higher the debt is (and the smaller the number of shares), the higher the EPS is and, therefore, the lower the debt is, the lower the PER is.

Lines 23 and 24 show the debt ratio calculated using book values and market values.

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4 The reader can deduce this expression from the following formulas, which correspond to the definition of ROA, ROE and PAT: ROA = NOPAT / (Dbv+Ebv) ROE = PAT / Ebv PAT = NOPAT-Kd Dbv(1-T)

5 This equation is obtained from NAxP = E and from NA = 5,000 - D/P. NA is the number of shares after repurchase.

The weighted average cost of capital. Line 25 shows the average cost of capital (WACC) using the market value debt ratio.

Line 26 shows the company's free cash flow, which is 60,000 dollars.

Line 27 shows the company's value, calculated by discounting the free cash flow at the WACC. Logically, it is the same as that calculated in line 13.

Implications. The most important results obtained from Table 1 are to be found in lines 13, 20 and 25. The company's optimal capital structure is that which *simultaneously*:

a) Maximizes the company's value (13),

b) Maximizes the share price (20), and

c) Minimizes the company's weighted average cost of capital (WACC)[6] (25).

Using the data given in Table 1, the optimal capital structure is attained with $150,000 of debt. Figure 1 shows how the company's optimal capital structure is determined: the company's value is highest and the WACC is lowest with $150,000 of debt (debt ratio = 30%). Figure 2 shows that the share price is also highest with $150,000 of debt (debt ratio = 30%).

Figure 1. Value of the company and WACC at different debt ratios

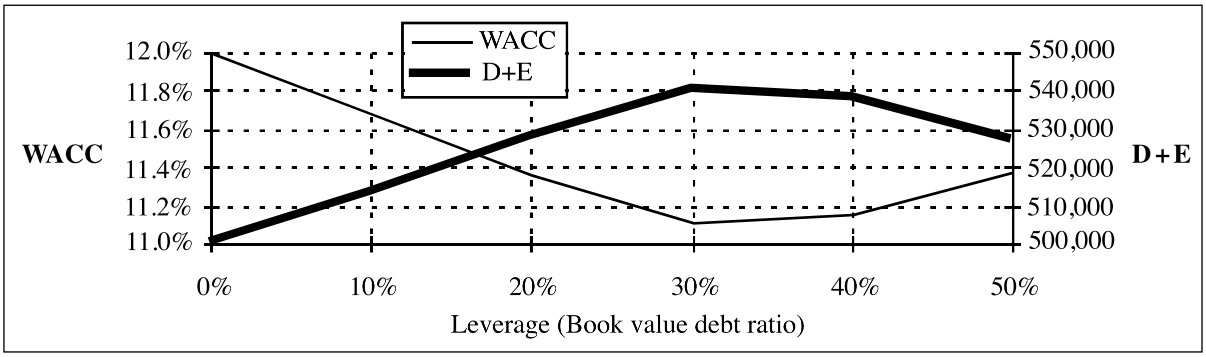
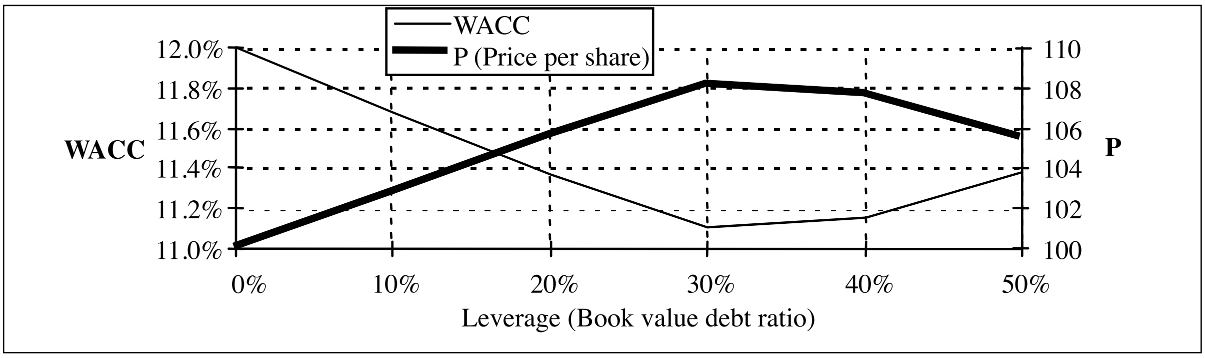


Figure 2. Price per share and WACC at different debt ratios



6 Because we are maximizing (D+E) in all three cases.

2. Critical analysis of the Harvard Business School technical note

The existence of an optimal structure with a debt ratio of 30% depends on debt and equity costs (lines 9 and 10) assumed by the note's author. The reader can see, for example, that with a graph in which Ke grows linearly with the debt ratio, the company's value increases at higher debt ratios. Likewise, if Ke were to be less than 14.4% (instead of 14.5%) for a debt level of $200,000, the optimal structure would be located at D= $200,000.

In this section, we will highlight certain inconsistencies in debt and equity costs (lines 9 and 10) assumed by the note's author from a number of viewpoints.

With respect to the cost of debt, the inconsistency is not the cost of debt (the bank can charge whatever interest it likes) but in assuming that the debt's cost is the same as its required return (or that the debt's value equals its nominal value).

2.1. Present value of the cash flows generated by the company and required return to assets

The sum of debt cash flow, equity cash flow and taxes at different debt levels is shown in line 2 of Table 2.

The tax risk in a perpetuity is the same as the equity risk[7]. Consequently, the discount rate that must be used to calculate the taxes' present value is Ke, as shown in line 28.

Table 2. Value of the cash flows generated by the company and required return to assets

	Book value of debt, D	**0**	**50,000**	**100,000**	**150,000**	**200,000**	**250,000**
5	Taxes	60,000	57,938	55,625	52,688	49,000	44,375
3	Debt cash flow (interest)	0	4,125	8,750	14,625	22,000	31,250
7	Equity cash flow (dividends)	60,000	57,938	55,625	52,688	49,000	44,375
2	Sum = cash flow generated by the firm = EBIT	120,000	120,000	120,000	120,000	120,000	120,000
28	Value of taxes. GOV = (5) / Ke	500,000	463,500	427,885	390,278	337,931	277,344
29	**D + E + GOV = (11) + (12) + (28)**	**1,000,000**	**977,000**	**955,769**	**930,556**	**875,862**	**804,688**
30	**Kassets = (2) / (29)**	**12.00%**	**12.28%**	**12.56%**	**12.90%**	**13.70%**	**14.91%**
31	*Δ Kassets*		*0.28%*	*0.27%*	*0.34%*	*0.81%*	*1.21%*

The company's total value (line 29) decreases with the leverage. There are only two explanations for this:

1. The cash flows generated by the company decrease with the leverage. In this case, this is not so, because it is assumed that the EBIT is $120,000/year, irrespective of debt.

2. The company's risk (and that of its assets) increases with the leverage. This causes this company's value to decrease with the leverage, as we shall see in the

[7] See chapter 17 of Fernández (2001 forthcoming), *Valuation and Shareholder Value Creation.*

following section. One explanation for this is that the providers of capital (shareholders, banks and capital markets) perceive a higher risk (more volatile with a higher likelihood of bankruptcy) in the company as a whole the more debt it includes in its capital structure.

The required return to assets (line 30) increases with the leverage[8], and increases much more when it goes above $150,000 (optimal structure). This sharp increase is the reason why an optimal structure exists.

2.2. Leverage costs

The expression *adjusted present value*, APV, by which the value of the levered company (D+E) is the sum of the value of the unleveraged firm (Vu) plus the discounted value of the tax shield (DT because our firm is a perpetuity) less the cost of leverage, is:

D+E = Vu + DT – Cost of leverage

As we know that Vu = 500,000 (line 16), we can find the value of the cost of leverage (line 32 of Table 3). Note that cost of leverage increases sharply when debt is increased from 150,000 to 200,000. The optimal structure appears just before the increase in the tax shield (line 34) becomes less than the cost of leverage (line 33).

Table 3. Cost of leverage

	Book value of debt, D	**0**	**50,000**	**100,000**	**150,000**	**200,000**	**250,000**
32	Cost of leverage	0	11,500	22,115	34,722	62,069	97,656
33	Δ Cost of leverage		11,500	10,615	12,607	**27,347**	**35,587**
34	Δ (DT)		25,000	25,000	25,000	**25,000**	**25,000**

2.3. Incremental cost of debt

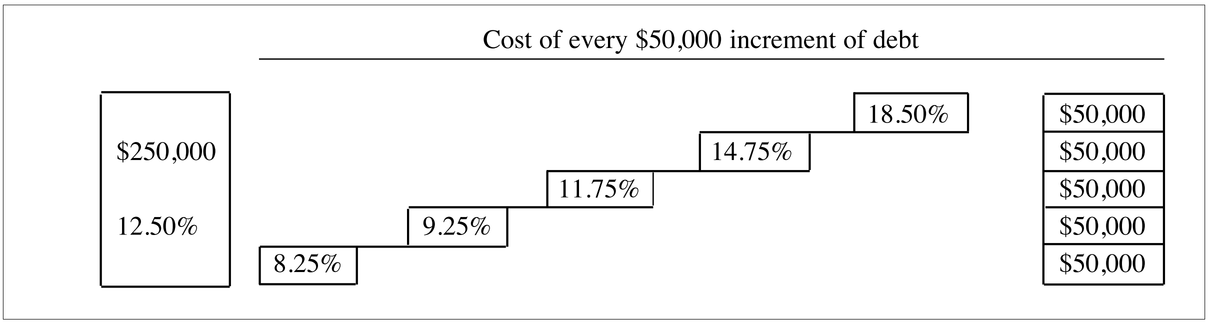
In this section, we will analyze the incremental cost of debt. Table 4 and Figure 3 show this analysis. It will be readily seen that the fact that $100,000 of debt have a cost of 8.75% means that the first $50,000 have a cost of 8.25% and the next $50,000 have a cost of 9.25%. It is a little surprising that the last two $50,000 increments have a cost of 14.75% and 18.5%, particularly considering that the required return to equity in the unlevered company is 12%.

Table 4. Incremental cost of debt

	Book value of debt, D	**0**	**50,000**	**100,000**	**150,000**	**200,000**	**250,000**
35	First 50,000		8.25%	8.25%	8.25%	8.25%	8.25%
36	Next 50,000			9.25%	9.25%	9.25%	9.25%
37	Next 50,000				11.75%	11.75%	11.75%
38	Next 50,000					14.75%	**14.75%**
39	Next 50,000						**18.50%**
40	Average		8.25%	8.75%	9.75%	11.00%	12.50%

[8] The required return to equity can also be obtained from the formula Kassets = [EKe + DKd (1-T)] / [E + D (1-T)]

Figure 3. Composition of the \$250,000 of debt, which has an overall cost of 12.5%



2.4. Required return to incremental equity cash flow

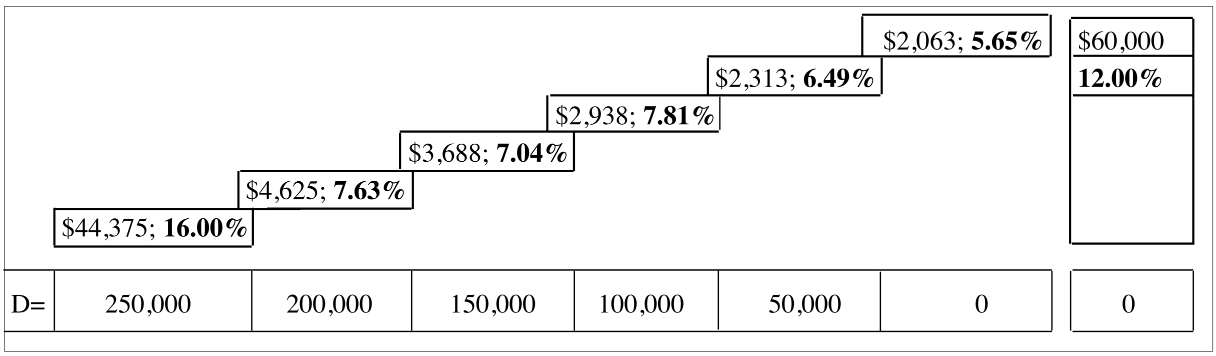
When the debt level is decreased, dividends increase and the shares' value grows. The required return to incremental equity cash flow is calculated in Table 5 and Figure 4 by performing an analysis similar to that performed with debt. The required return to incremental equity cash flow is calculated as follows. ED is the shares' value when the company has a debt D. With this debt level, the dividends are Div. When the debt level is decreased, dividends increase to (Div + ΔDiv) and the shares' value increases from E_D to E_{D-} . Ke^{INC} is the required return to the additional equity. The following equation must be met:

$$Ke^{INC} = \Delta Div / (E_{D-} - E_D)$$

Table 5. Required return to incremental equity cash flow

	Book value of debt, D	**0**	**50,000**	**100,000**	**150,000**	**200,000**	**250,000**
10	Ke	12.00%	12.50%	13.00%	13.50%	14.50%	16.00
	Required return to incremental equity cash flow (from right to left):						
41	Incremental equity cash flow (ΔDiv)	2,063	2,313	2,938	3,688	4,625	
42	Required return to incremental equity cash flow	***5.65%***	***6.49%***	***7.81%***	***7.04%***	***7.63%***	

Figure 4. Required return to the incremental equity cash flow when the debt level is decreased



Note that the required incremental return first falls from 7.63% to 7.04%, then increases to 7.81% and then falls again. The increase from 7.04% to 7.81% is an error because the required incremental return should decrease as the leverage decreases.

2.5. Difference between Ke and Kd

Table 7 shows that the difference between Ke and Kd decreases for debt levels above 100,000 dollars.

Table 7. Difference between Ke and Kd

	Book value of debt, D	0	50,000	100,000	150,000	200,000	250,000
43	Ke - Kd	4.00%	4.25%	4.25%	3.75%	3.50%	3.50%
44	Ke - Kd (1-T)	8.000%	8.375%	8.625%	8.625%	9.000%	9.750%

2.6. Price per share for different debt levels

Table 8 shows the price per share if the company's leverage goes from the debt-free situation to the desired level of leverage: it is the same as the price per share (line 20) of Table 1. Line 45 of Table 8 shows the price per share if the company's leverage is increased stepwise: first, \$50,000 of debt are added, then another \$50,000 and so on.

Table 8. Price per share for each step of debt level

	Book value of debt, D	0	50,000	100,000	150,000	200,000	250,000
20	Price per share leveraging the firm from D=0 until final leverage						
			102.70	105.58	**108.06**	107.59	105.47
45	Price per share leveraging the firm in steps of \$50,000 each						
			102.70	108.62	**113.38**	106.20	97.77

2.7. Adding the possibility of bankruptcy to the model

This model allocates a probability to the likelihood that the company will go bankrupt and there will be no more dividend or interest payments. In the extreme case that the bondholders recover none of their investment, the value of the interest payments they will receive is:

$I_{t+1} = I_t$ with a probability $p_c = 1 - p_q$

$0 = D_{t+1}$ with a probability p_q

In this case, the debt's value at t=0 is:

$D_0 = I\,(1 - p_q) / (Kd + p_q)$. Kd is the required return on debt without leverage costs

Isolating the probability of bankruptcy, we obtain:

$p_q = (I - D_0\ Kd)/\ (I + D_0)$

From the shareholders' viewpoint, the value of the dividends they will receive is:

$Div_{t+1} =$ Div_t with a probability $p_c = 1 - p_q$

$0 = E_{t+1}$ with a probability p_q

In this case, the shares' value at t=0 is:

$E_0 = Div\ (1 - p_q)\ /\ (Ke + p_q)$. Ke is the required return to equity without leverage costs.

Isolating the probability of bankruptcy, we obtain:

$p_q = (Div - E_0\ Ke)/\ (Div + E_0)$

Table 8b. Probability of bankruptcy of debt and equity

	Book value of debt, D	**0**	**50,000**	**100,000**	**150,000**	**200,000**	**250,000**
46	Pq (debt)	0.000%	0.045%	0.307%	0.980%	1.820%	2.810%
47	Pq (shares)	0.000%	0.266%	0.517%	0.727%	1.328%	2.294%
	Kd if Pq = 0	***8.00%***	***8.20%***	***8.42%***	***8.67%***	***8.98%***	***9.34%***
	Ke if Pq = 0	***12.00%***	***12.20%***	***12.42%***	***12.67%***	***12.98%***	***13.34%***

Table 8b shows that the required returns to debt and equity assume that the probability of bankruptcy of debt exceeds that of the equity at debt levels greater than 150,000, which is absurd.

Upon performing a similar analysis with the entire company (debt, equity and taxes), the annual expected cash flow for all three is constant, irrespective of the leverage, and is equal to \$120,000 (see Table 2). Table 3 shows these flows' present value. The addition of the probability of bankruptcy (a total bankruptcy in which neither the bondholders nor the shareholders nor the State can recover anything) would mean that the expected value of the cash flow for the next period would be:

\$120,000 with a probability $p_c = 1 - p_q$

$0 = E_{t+1} + D_{t+1} + GOV_{t+1}$ with a probability p_q

For each level of leverage,

$E_0 + D_0 + GOV_0 = 120{,}000\ (1 - p_q)\ /\ (Ku + p_q)$

The probability of total bankruptcy gives:

Debt	**0**	**50,000**	**100,000**	**150,000**	**200,000**	**250,000**
D+E+GOV	1,000,000	976,992	955,770	930,548	875,862	804,688
Equity cash flow, taxes and debt cash flow	120,000	120,000	120,000	120,000	120,000	120,000
Pq (firm) if Ku=12%	**0.00%**	**0.25%**	**0.49%**	**0.79%**	**1.50%**	**2.53%**

It can be seen that the probability of bankruptcy almost doubles when debt level is increased from $150,000 to $200,000.

2.8. Ke and Kd if there are no leverage costs

If we assume that Ku = 12% (the assets' risk does not change with these debt levels and, therefore, there are no leverage costs), line 9' of Table 9 shows the Kd that is obtained after applying the following formula[9]:

$$Kd = R_F + \frac{D\ (1 = T\ (Ku - R_F)}{E + D\ (1 - T)}$$

Table 9. Valuation without leverage costs

1	**Book value debt level (leverage)**	**0%**	**10%**	**20%**	**30%**	**40%**	**50%**
9	**Cost of debt: r**	**8.00%**	**8.25%**	**8.75%**	**9.75%**	**11.00%**	**12.50%**
9'	**Required return on debt: Kd**	**8.00%**	**8.20%**	**8.42%**	**8.67%**	**8.98%**	**9.34%**
10	**Required return on equity: Ke**	**12.00%**	**12.20%**	**12.42%**	**12.67%**	**12.98%**	**13.34%**
11	Market value of debt, D. (3)/(9')	0	50,298	103,970	168,600	244,990	334,635
12	Market value of equity, E. (7)/(10)	500,000	474,851	448,015	415,700	377,505	332,683
13	Market value of the firm. (11)+(12)	500,000	525,149	551,985	584,300	622,495	667,317
19	Number of shares outstanding, NA	5,000	4,524	4,088	3,674	3,268	2,855
20	Price per share, P (12)/(19)	100	104.970	109.603	113.140	115.501	116.537
24	Market value debt level (leverage). (11)/(13)	0.00%	9.58%	18.84%	28.85%	39.36%	50.15%
25	Weighted average cost of capital (WACC)	12.00%	11.43%	10.87%	10.27%	9.64%	8.99%
28	Value of taxes. GOV = (5) / Ke	500,000	474,851	448,015	415,700	377,505	332,683
29	D + E + GOV = (11) + (12) + (28)	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000

In all cases, r > Kd, which is why the debt's value is greater than its nominal value. Similarly, line 10 shows the Ke obtained after applying the following equation:

$$Ke = Ku + Kd - R_F$$

[9] The reader can verify the deduction of this and the following equations in chapter 17 of Fernández (2001 forthcoming), *Valuation and Shareholder Value Creation.*

Note that in this case:

- There is no optimal structure. The company's value (line 13) increases with the debt ratio
- Debt's value is substantially higher than the nominal value
- The difference between Ke and Kd is constant and equal to 4%

2.9. Ke and Kd with leverage costs

Table 10 assumes the existence of leverage costs, shown by the use of the reduced formula for the leveraged beta, which is equivalent to using formula [23.31] for the required return to equity:

$$Ke = Ku + (D/E)\ (Ku – R_F)$$

This is equivalent to assuming that the required return to assets increases with the leverage (line 30).

Table 10. Valuation with leverage costs

1	**Book value debt level (leverage)**	**0%**	**10%**	**20%**	**30%**	**40%**	**50%**
9	**Cost of debt: r**	**8.00%**	**8.25%**	**8.75%**	**9.75%**	**11.00%**	**12.50%**
9'	**Required return on debt: Kd**	**8.00%**	**8.22%**	**8.48%**	**8.86%**	**9.41%**	**10.27%**
10	**Required return on equity: Ke**	**12.00%**	**12.43%**	**12.96%**	**13.72%**	**14.83%**	**16.54%**
11	Market value of debt, D. (3)/(9')	0	50,210	103,174	165,074	233,685	304,337
12	Market value of equity, E. (7)/(10)	500,000	466,076	429,150	384,038	330,438	268,346
13	Market value of the firm. (11)+(12)	500,000	516,286	532,324	549,112	564,123	572,683
19	Number of shares outstanding, NA	5,000	4,516	4,055	3,596	3,115	2,588
20	Price per share, P (12)/(19)	100	103.21515	105.83007	**106.80758**	106.08768	103.66923
21	Earnings per share (EPS). (6)/(19)	12	12.8306	13.7173	14.6533	15.7315	17.1432
22	Price-earnings ratio, PER	8.33333	8.04446	7.71506	7.28898	6.74364	6.04724
24	Market value debt level (leverage). (11)/(13)	0.00%	9.73%	19.38%	30.06%	41.42%	53.14%
25	Weighted average cost of capital (WACC)	12.00%	11.62%	11.27%	10.93%	10.64%	10.48%
28	Value of taxes. GOV = (5) / Ke	500,000	466,076	429,150	384,038	330,438	268,346
29	D + E + GOV = (11) + (12) + (28)	**1,000,000**	**982,362**	**961,475**	**933,150**	**894,562**	**841,029**
30	**Kassets = (2) / (29)**	**12.00%**	**12.22%**	**12.48%**	**12.86%**	**13.41%**	**14.27%**
31	Δ *Kassets*		*0.22%*	*0.27%*	*0.38%*	*0.55%*	*0.85%*
32	Cost of leverage	0	8,819	19,263	33,425	52,719	79,486
33	Δ Cost of leverage		8,819	10,444	14,163	19,294	26,766
34	Δ (DT)		25,105	26,482	30,950	34,305	35,326
	Required return to incremental equity cash flow (from right to left):						
41	Incremental equity cash flow (DDiv)	2,063	2,313	2,938	3,688	4,625	
42	Required return to incremental equity cash flow	***6.08%***	***6.26%***	***6.51%***	***6.88%***	***7.45%***	
43	Ke - Kd	**4.00%**	**4.22%**	**4.48%**	**4.86%**	**5.41%**	**6.27%**
44	Ke - Kd (1-T)	**8.00%**	**8.32%**	**8.72%**	**9.29%**	**10.12%**	**11.40%**
45	Price per share. Incremental repurchase	0	103.215	108.581	**108.818**	103.985	95.006
46	Pq (debt)	0,000%	0.013%	0.060%	0.170%	0.397%	0.843%
47	Pq (shares)	0,000%	0.204%	0.483%	0.919%	1.610%	2.744%

Kd is calculated using the formula:

$$Kd = R_F + [D (1\text{-}T) (Kassets - R_F)] / [E + D (1\text{-}T)]$$

In this case, as the leverage is increased, the WACC decreases and the company's value increases. The maximum price per share occurs at N = $150,000.

Note that lines 31, 33, 34, 42, 43, 46 and 47 no longer have the inconsistencies identified in previous sections.

2.10. Influence of growth on the optimal structure

If a perpetual growth g is applied to the data in Table 1 and it is assumed that the first year's investment in net fixed assets and WCR (working capital requirements) is $500,000 x g (all $500,000 of the initial outlay is invested in WCR and fixed assets), for any growth level the optimal structure continues to be a debt level of $150,000.

3. Boeing's optimal capital structure according to Damodaran

Damodaran[10] offers a similar approach to that of the Harvard Business School example analyzed, but applies it to a real company (Boeing in 1990) and assumes a constant cash flow growth of 8.86%.

Damodaran's calculations are summarized in Table 11. According to him, Boeing's optimal structure[11] is attained with a debt ratio of 30% (the debt ratio is calculated from the equity's book value). One problem with Table 11 is that the value of the firm (D+E) for debt ratios above 70% is less than the value of debt, which implies a negative value for equity. Of course, this does not make any sense.

Table 11. Optimal capital structure for Boeing (million dollars). March 1990

Leverage (book value)	Value of the firm	Value of debt	Value of equity	Cost of debt (After Tax)	Cost of debt	Incremental debt	Cost of incremental debt
10%	17,683	1,646	16,037	6.40%	9.70%	1,646	**9.70%**
20%	18,968	3,292	15,676	6.93%	10.50%	1,646	**11.30%**
30%	**19,772**	4,938	14,834	7.59%	11.50%	1,646	**13.50%**
40%	18,327	6,584	11,743	9.24%	14.00%	1,646	**21.50%**
50%	17,657	8,230	9,427	9.90%	15.00%	1,646	**19.00%**
60%	14,257	9,876	4,381	11.72%	16.50%	1,646	**24.00%**
70%	10,880	11,522	**–642**	13.90%	18.00%	1,646	**27.00%**
80%	9,769	13,168	**–3,399**	14.42%	18.00%	1,646	**18.00%**
90%	8,864	14,814	**–5,950**	14.81%	18.00%	1,646	**18.00%**

Source: *Damodaran on valuation*, pp. 159.

[10] See Damodaran (1994), *Damodaran on valuation*. pp. 157-164 and 167-169.

[11] In March 1990, the book value of Boeing's debt stood at $277 million and the market value of its equity was $16.182 billion. Consequently, the company's value, according to Damodaran, was $16.459 billion (0.277+16.182).

The last column of Table 11 shows the cost of the assumed debt increments. It can be seen that increasing the debt by $1.646 billion to take the debt ratio from 30% to 40% implies contracting that debt at 21.5%, which is an enormous figure. Stranger still is the finding that the next debt increment (which has a higher risk) is cheaper: it costs 19%.

Table 12 shows the forecast income statements and cash flows for Boeing with different leverages.

Table 12. Optimal capital structure for Boeing. Capital structure, income statements and cash flows according to Damodaran (million dollars). March 1990

1	**D/(D+E) book value**	**0%**	**10%**	**20%**	**30%**	**40%**	**50%**	**60%**	**70%**	**80%**	**90%**
2	(D/E)book value	0%	11%	25%	43%	67%	100%	150%	233%	400%	900%
3	Debt (D)	0	1,646	3,292	4,938	6,584	8,230	9,876	11,522	13,168	14,814
4	Kd	9.7%	9.7%	10.5%	11.5%	14.0%	15.0%	16.5%	18.0%	18.0%	18.0%
5	Taxes	34%	34%	34%	34%	34%	34%	28.96%	22.76%	19.91%	17.70%
6	Beta unleveraged	0.94	0.94	0.94	0.94	0.94	0.94	0.94	0.94	0.94	0.94
	Income statement of year 0										
7	Margin	2,063	2,063	2,063	2,063	2,063	2,063	2,063	2,063	2,063	2,063
8	Depreciation	675	675	675	675	675	675	675	675	675	675
9	Interest**	0	160	346	568	922	1,235	1,630	2,074	2,370	2,667
10	Profit before taxes	1,388	1,228	1,042	820	466	154	–242	–686	–982	–1,279
11	Taxes (34%)	472	418	354	279	159	52	–82	–233	–334	–435
12	**Profit after taxes**	**916**	**811**	**688**	**541**	**308**	**101**	**–159**	**–453**	**–648**	**–844**
13	+ depreciation	675	675	675	675	675	675	675	675	675	675
14	– investment in fixed assets	800	800	800	800	800	800	800	800	800	800
15	– investment in working capital	0	0	0	0	0	0	0	0	0	0
16	+ increase of debt	0	146	292	438	583	729	875	1,021	1,167	1,313
17	**Equity cash flow, ECF**	**791**	**832**	**855**	**854**	**766**	**705**	**591**	**443**	**393**	**344**
18	**Free cash flow, FCF**	**791**	**791**	**791**	**791**	**791**	**791**	**791**	**791**	**791**	**791**
19	g (growth)	8.86%	8.86%	8.86%	8.86%	8.86%	8.86%	8.86%	8.86%	8.86%	8.86%

Source: *Damodaran on valuation*, pp. 167-169.

Table 13. Optimal capital structure for Boeing. Valuation according to Damodaran. Data in million dollars. March of 1990

	D/(D+E) book value	**0%**	**10%**	**20%**	**30%**	**40%**	**50%**	**60%**	**70%**	**80%**	**90%**
20	Leveraged beta	0.94	1.0089	1.0951	1.2059	1.3536	1.5604	1.9417	2.6341	3.9514	7.9026
21	Market risk premium	5.5%	5.5%	5.5%	5.5%	5.5%	5.5%	5.5%	5.5%	5.5%	5.5%
22	R_F	9.0%	9.0%	9.0%	9.0%	9.0%	9.0%	9.0%	9.0%	9.0%	9.0%
23	**Ke' (calculated with book value)**	**14.17%**	**14.55%**	**15.02%**	**15.63%**	**16.44%**	**17.58%**	**19.68%**	**23.49%**	**30.73%**	**52.46%**
24	**WACCbv**	**14.17%**	**13.73%**	**13.40%**	**13.22%**	**13.56%**	**13.74%**	**14.90%**	**16.78%**	**17.68%**	**18.58%**
25	(D+E) = NPV (FCF;WACC)	16,218	17,667	18,950	19,753	18,312	17,643	14,247	10,875	9,764	8,861
26	**–D = E1**	**16,218**	**16,021**	**15,658**	**14,815**	**11,728**	**9,413**	**4,371**	**–647**	**–3,404**	**–5,953**
27	**E2 = NPV (ECF; Ke)**	**16,218**	**15,911**	**15,095**	**13,724**	**10,995**	**8,805**	**5,942**	**3,298**	**1,958**	**858**

Source: *Damodaran on valuation*, pp. 167-169.

Table 13 contains the valuation of the cash flows and is the origin of the numbers in Table 4. Another error in Table 13 is that lines 26 and 27 are only the same for the unlevered company. Why is this so? Basically, for two reasons:

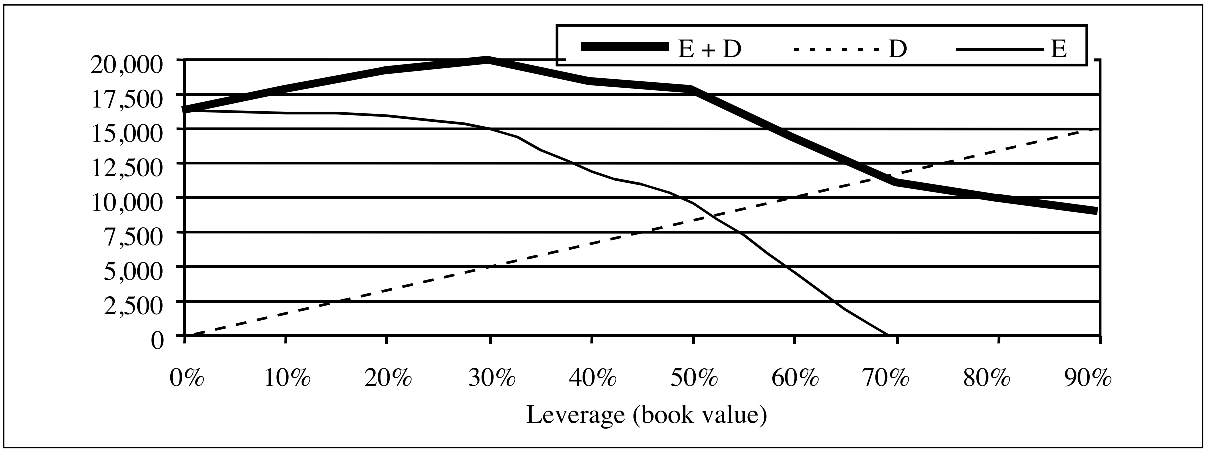
1. Damodaran calculates the WACC using book values in the weighting, instead of market values.

2. Damodaran calculates the interest to be paid in year 0 (line 9 of Table 12) by multiplying the debt in year 0 (line 3) by the cost of debt (line 4). In order to obtain a correct valuation, the interest for year 0 should be calculated by multiplying the debt in the previous year (year -1) by the cost of debt. This affects equity cash flow.

We leave the reader to verify that when these two adjustments are made, lines 26 and 27 of Table 13 match. The main lines that change are the following:

	D/(D+E) book value	**0%**	**10%**	**20%**	**30%**	**40%**	**50%**	**60%**	**70%**	**80%**	**90%**
9	Interest**	0.0	146.7	317.5	521.7	846.7	1,134.0	1,496.9	1,905.2	2,177.3	2,449.5
16	+ Increase of debt	0.0	134.0	267.9	401.9	535.9	669.8	803.8	937.8	1,071.7	1,205.7
17	Equity cash flow, ECF	791.1	828.2	849.4	848.7	768.1	712.5	601.4	413.3	314.6	207.1

Figure 5. Boeing according to Damodaran. Value of the firm (D+E), debt and equity, for different debt ratios



Key concepts

Optimal capital structure, required return to incremental equity cash flow, incremental cost of debt, required return on debt (Kd), required return to equity (Ke), adjusted present value (APV), return on assets (ROA), return on equity (ROE), weighted average cost of capital (WACC), free cash flow (FCF).

References

Damodaran, Aswath (1994), *Damodaran on Valuation*, John Wiley and Sons, New York.

Fernández, P. (2001 forthcoming), *Valuation and Shareholder Value Creation*.

Fruhan, W. E.; W. C. Kester; S. P. Mason; T. R. Piper; and R. S. Ruback (1992), "Note on the Theory of Optimal Capital Structure", in the book *Case Problems in Finance*, Irwin, 10th edition.