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A SIMULATION MODEL OF OPTIMAL INCOME TAX FUNCTIONS WITH A STABILIZATION CONSTRAINT

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Abstract

The aim of this paper is to present an optimal linear income taxation model to use in computing the optimal values of the tax function considered.

The main characteristic of the model is the maximization of the social welfare function by the public sector, subject to two constraints; a revenue constraint and a stabilization objective constraint. This second constraint has not yet been considered in optimal tax theory, but it has an unquestionable meaning: income tax not only has effects on the distributional activities of the government, but also on its stabilization policies.

For this reason, it is worthwhile considering this government's function in optimal income tax design. The analysis suggests that the range of variation of optimal tax rates is narrowed when the stabilization constraint is introduced.

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1. Introduction

Optimal tax theory provides one of the most serious contributions to the study of modern fiscal reform. Highly interesting theoretical results have been obtained for tax rates (their structure and variation) and the suitable basis[1] assessment through studies of both income taxation and indirect taxation.

The purpose of this paper is to introduce a specific macroeconomic goal in the classical optimal taxation models and evaluate how the results obtained under this new set of conditions[2] differ from those obtained from the conventional model. In addition, we apply this new model to simulate the values adopted by the linear taxation function, taking into account certain parameters of the Spanish economy.[3]

These parameters are not precise values but estimates on which we will base the appropriate sensitivity analysis. Consequently, we propose carrying out an exhaustive sensitivity analysis. This lack of information should not cause any surprise, as the parameters being handled are very concrete and, with the exception of a few studies, knowledge about them is scarce in almost all countries. Our approach has been the same as that of N. Stern (1976), with the exception that Stern uses, as a reliable elasticity of substitution value, the one obtained by Ashenfelter and Heckman (1973) for the United States, while we must also perform a sensitivity analysis for the value given to the elasticity of substitution. Perhaps it is for this reason that our work is more closely related with that of J.A. Mirrlees (1971) than with that of M. Tuomala (1984) or even Stern, although it must be pointed out that Mirrlees studies the properties of a general income taxation (and not a linear income taxation) as we do, and his sensitivity analysis is more limited than ours.

There are main differences between this model and those mentioned above. First, it offers a highly exhaustive sensitivity analysis of tax parameters, based on the values taken by the various behavioral function parameters. This sensitivity analysis will be simulated for a wide

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[1] See Sheshinski (1971), Mirrlees (1971), Atkinson-Stiglitz (1973), Sandmo (1976), Stern (1976) and Tuomala (1984).

[2] This constraint has been considered in previous literature. For more details see Canals (1988), Chapter VI.

[3] See Ahijado (1983) and Malo de Molina (1984).

range of values for the following parameters: "Elasticity of substitution between consumption and leisure," "inequality aversion," "share of expenditure in pure public goods," "individual utility function argument weighting coefficient" and "standard deviation of the distribution density of abilities function." Stern, for example, only performs a sensitivity analysis for the first three parameters and within a very limited range of values. Mirrlees, on the other hand, analyzes the effect of a variation in the standard deviation of the above-mentioned density function and the inequality aversion parameter; but he avoids considering any other parameters, because, among other reasons, his model does not allow him to. Finally, Tuomala permits variations in the inequality aversion coefficient but keeps the remaining parameters constant. Therefore, although our model is based on those of the authors quoted, it widens the range of results and allows for certain structural data concerning the Spanish economy.

Another major difference is that, in our model, the taxation parameters will be determined assuming that, in the first place, the public sector fulfils its revenue constraint, and, in the second place, especially using the marginal tax rate, a flexible tax structure is attained, enabling the public sector to achieve certain results of the economic macromagnitudes.[4] Finally, we want to compare the optimal tax rates in both cases, that is, in the classical model and in a model with macroeconomic goals.

The intertemporal approach to fiscal policy analysis can be very useful; nevertheless, we avoid this approach in this paper because it would make the computational analysis we carry out in the next section more complex, without adding any substantial explanatory elements to our main question, i.e., how do optimal tax results differ from the standard model when the government has some kind of stabilization policies, in addition to pure redistribution objectives? Besides, we are interested in comparing the results of this new hypothesis with those obtained in the previous models, which don't adopt the intertemporal approach.

The theoretical model we use is described in the next section. This model consists of a social welfare function which the government intends to optimize; a revenue constraint on the public sector; a constraint imposed by the stabilization goals pursued by the government; a CES-type individual utility function whose arguments are initially after-tax income and leisure and which, in a second phase, will be expanded to also include the supply of pure public goods; a revenue constraint on any individual; an ability distribution function among the population; and, finally, we will use an aggregate production function with labor as the sole production factor measured in units of efficiency. The lack of distinction between types of labor (and the exclusion of capital as a production factor) is perhaps this model's greatest limitation.[5]

Section 3 contains a brief description of the hypothesis and the computation procedures used. The results obtained are presented and discussed in section 4.

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[4] See Turnovsky (1977), Barro (1986) and Canals (1987).

[5] Anyway, this simplification is a common point in many papers on this topic.

2. An Optimal Linear Income Taxation Model

2.a. Consumer behavior

The model described below, with a few minor differences, is the classic model used in optimal income taxation. The utility of the ith individual depends on his after-tax income (y_{di}) and leisure ($1-l_i$) and it follows the CES-type functional form, i.e.:

$$(1)\quad U_i = \{ a_1\ y_{di}^{\beta} + a_2\,(1-l_i)^{\beta} \}^{\frac{1}{\beta}}$$

where $a_1 + a_2 = 1$. The advantages of using this utility function, and not Mirrlees' linear log function, for example, are twofold. Firstly, the CES function accepts values for the elasticity of substitution that are different from one and, secondly, it barely alters responses as regards the decision to offer more or fewer hours of labor as a result of taxes.[6]

In equation (1) above, parameters a_1 and a_2 express the weight that each individual gives to income and leisure. Parameter β determines the value of the elasticity of substitution between income and leisure, according to the equation

$$(2)\quad e = \frac{1}{1-\beta}$$

where e is the elasticity of substitution. The variable y_{di} expresses the after-tax income of the ith individual while l_i indicates the amount of time spent working and, therefore ($1-l_i$), the amount of time spent on leisure ($0 \leq li \leq 1$).

Each individual will try to optimize that utility function with the following revenue constraint (pre-tax):

$$(3)\quad y_i = y_{di} = w_i\, l_i$$

where w_i is the wage rate of the ith individual. This rate is assumed to perfectly reflect the ith individual's ability and its distribution among the population can be estimated from the normal logarithmic density function:

$$(4)\quad f(w) = \frac{1}{w \angle \sqrt{2\pi}}\ e^{\left\{ \frac{-(\ln\ w - \mu)^2}{2\angle^2} \right\}}$$

where μ and $\angle$ are the mean and standard deviation of the distribution. In the future, we will use the standardized function f (w) which implies that

$$(5)\int_0^{\infty} f(w)dw = 1$$

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[6] For a discussion on the advantages of one application of the CES, see Zabalza, Pissarides and Barton (1980).

This affects some magnitudes which are obtained by integrating w-dependent functions, as these magnitudes will express "per capita" values. A final comment on the wage rate wi: this rate will not be expressed in monetary units but in efficiency units; in general, the wage rate of one efficiency unit is one.

When a linear income tax function of the type

$$(6) \quad = -y_0 + tw_i\, l_i$$

is introduced, where y_0 is a guaranteed income and t is the constant marginal rate, the equation for the ith individual's revenue becomes:

$$(7) \quad y_{di} = y_i - T = y_0 + (1-t)\, w_i\, l_i$$

To obtain the conditions for an optimal behavior by the ith consumer, equation (7) will be substituted into the utility function, giving

$$(8) \quad U_i = \{a_1 (y_0 + (1-t)\, w_i l_i)^{\beta} + a_2 (1 - l_i)^{\beta}\}^{\frac{1}{\beta}}$$

Upon deriving U_i with respect to l_i, providing that the revenue constraint is fulfilled, making the derivative equal to zero, and transferring l_i to the left-hand side of the equation, it is found that the condition for a first order maximum is:

$$(9) \quad l_i = \frac{\left\{\frac{a_2}{a_1}\frac{1}{(1-t)\, w_i}\right\}^{\frac{1}{\beta-1}} - y_0}{(1-t)\, w_i + \left\{\frac{a_2}{a_1}\frac{1}{(1-t)\, w_i}\right\}^{\frac{1}{\beta-1}}}$$

This equation expresses the supply of labor by the ith individual when his behavior is optimal.

2.b. The public sector

For its part, the problem of the public sector consists of choosing values for the parameters t and y0 which maximize and additive-type social welfare function as follows:

$$(10) \quad W = \frac{1}{\varepsilon} \int_0^{\infty} U_i^{\varepsilon} (y_{di}, l_i)\, f(w)\, dw$$

provided that the revenue constraint on the public sector is fulfilled. This constraint can be expressed as follows:

$$(11) \quad \int_0^{\infty} t\, w_i\, l_i\, f(w)\, dw = y_0$$

where the expression

$$(12) \quad \int_0^{\infty} w_i \, l_i \quad f(w) \ dw$$

in the absence of the capital factor and production factors other than labor, expresses the national "per capita" income, which will be represented by R.

If, in addition, the public sector pursues a macro-economic related goal using fiscal policy, it is possible to formulate a welfare function optimization problem dependent not only on revenue constraints, but also on the constraint imposed by the macro-economic goal pursued. If this goal is to reach certain GDP level (for instance, full employment GDP) this constraint can be expressed as follows:[7]

$$(13) \quad y^* = \int_0^{\infty} w\, l^* \ f(w) = \int_0^{\infty} y_0^* + (1-t^*)\, w_2 \, l_i + G_0^*$$

where the left-side integral is the aggregate supply – under the condition defined in this section – and the right-side expresses the aggregate demand. Y0* , t * and G0 * are, respectively the optimal values of the government payments, the marginal income tax and the government spending (different from y0, if exists). With the management of these instruments, the government can reach the target level of GNP.

It is evident that a more innovating approach to the modern fiscal policy would have made use of an intertemporal model (7), with the classical equations:

$$(14) \quad U_i = U\,(C_0\,,\, G_0) + \alpha\, V\, U\,(C_1\,,\, G_1)$$

and

$$(15) \quad G = G_0 + \alpha\, G_1 = T_0 + \alpha\, T_1 - (1+ i_{-1})\, B_{-1}$$

where C is the private consumption, G the government spending, α the discount factor, T the taxes raised, B_{-1} the debt issued by the government previously and i_{-1} the interest rate of this debt issue.

It is simple to recognize that equation (14) expresses the standard individual's utility function over two periods, 0 and 1, and equation (15) represents the government solvency constraint for the same two periods. The α parameter denotes the present value factor.

Nevertheless, as mentioned, we leave out this intertemporal approach to the tax policy because we don't want to analyze the properties of the optimum income tax, but rather to compare it when we introduce other objectives, as already described, with the classical optimal income tax function.

[7] Frenkel and Razin (1987) provide an excellent framework to analyze the effects of the fiscal policy using intertemporal macroeconomics.

In the following section, we will see the differences in marginal tax rates and lump-sum transfers when we consider the problem of optimizing a social welfare function that is dependent only on revenue constraints and when we add to this the constraint derived from the stabilization goals that the government has set itself.

What is the nexus between the optimal behavior of any individual and that of the public sector? The answer is found in the previous equation (9) which expresses the supply of labor derived from the optimal behavior for the individual under study. In fact, the public sector maximizes the sum of the utility of all individuals in society on the basis of the hypothesis that each one has optimized his individual utility beforehand. This is done by substituting variable l_i into the equation for U_i – equation (8) – instead of its expression in equation (9) and later substituting the new formulation of U_i in (10).

A final consideration in the description of the theoretical model that we will use concerns the modification of the individual utility function when the supply of pure public goods enjoyed by the ith individual are introduced into it for the purposes of argument. These are goods that the public sector offers and whose financing requires the levying of taxes.[8] If P is the supply of public goods – their price is expressed in terms of the price of private goods – and g is the share of expenditure in public goods of the national income, we can write an equation as follows:

$$(16)\quad P = g\,R$$

The introduction of public goods in the U_i utility function – equation (1) – converts this function into:

$$(17)\quad U_i = \{a_1\, y_{di}^{\beta} + a_2\,(1-l_i)^{\beta} + a_3\,P^{\beta}\}^{\frac{1}{\beta}}$$

where the sum of a_1, a_2, and a_3 must be equal to one. If $a_3 = 0$, that is to say, the supply of public goods does not provide any utility for the individual, equation (1) will again apply. The new condition for a first order maximum is:

(9’)

$$l_i = \frac{\left\{\frac{1}{(1-t)\,w_i\ a_1}\quad(a_2 - a_3)\right\}^{\frac{1}{\beta-1}} - y_0}{(1-t)\,w_i + \left\{\frac{a_2}{a_1}\ \frac{1}{(1-t)\ w_i}\right\}^{\frac{1}{\beta-1}}}$$

Finally, the introduction of public goods also modifies the public sector budget equation which will now be written as follows:

$$(18)\quad \int_0^{\infty} t\ w_i\,l_i\ f(w)\ dw = y_0 + gR$$

[8] See Moreh (1983).

Grouping together the terms and substituting R by its expression in (12), the revenue equation becomes:

$$(19) \quad y_0 = (t - g)\ R$$

or

$$(20) \quad y_0 + gR = tR$$

The final expression has an immediate meaning: the sum of the minimum income guaranteed to each individual by linear income taxation and expenditure by the public sector on the supply of public goods must be equal to the amounts collected solely by income taxation; we therefore assume that there are no other taxes.

3. Description of the Hypothesis and Computation of the Optimal Tax Rates

This section describes the mathematical methods and hypothesis used to find the optimal marginal tax rates. As in the classic optimal taxation models, the purpose is to determine marginal tax rates that maximize the value of the social welfare function for the community.

For the moment we will not consider the government goals concerning stabilization policy. We assume that the social welfare function, the public sector's budget equation, the ith individual's utility function, the ith individual's budget equation and the labor supply that optimizes the behavior of the ith individual have been specified. The optimal tax rate computation model is based on simulating what the optimal value of the different variables calculated would be if the model parameters were to have a specific combination of values. In each case, we will work with initial parameter values that coincide with certain estimates made for the Spanish economy, and furthermore we will perform an exhaustive sensitivity analysis of the variables in relation to the variations of a particular parameter. Occasionally, the base point value will be that estimated in another country or in other circumstances.

We have used the normal logarithmic density function to reflect the distribution of the ability parameter among the general population. We will initially take the values of the μ and $\angle$ parameters obtained for Great Britain by H.I. Lydall (1968), i.e., $\mu = -1$ and $\angle = 0.39$. This is the hypothesis used in Mirrless, Stern and Moreh's optimal taxation models with numerical computation. However, we will implement further on a standard deviation $\angle = 1$ in order to analyze the effect of an increase in the standard deviation of the "ability" parameter on the optimal rates. In some studies in the Spanish economy, the value $\mu = -1$ has been obtained (e.g., Canals,1988).

In the case of the social welfare function, the relevant parameter is ε, the elasticity of marginal utility of income. When ε falls, the marginal utility of income decreases at a faster rate and the consequent loss of utility (which is usually interpreted as the cost of inequality in the distribution of income) increases. Thus, the smaller ε is, the greater the marginal tax rate will be. This parameter indicates society's inequality aversion. Stern[9] has calculated the approximate values for this parameter in Great Britain and the United States, using a comparison of the marginal utilities of income before and after tax. The values obtained are –1 and –0.5 respectively. We use possible values for ε, among which are included the above-mentioned values. Mirrlees uses as his base-point value $\varepsilon = 0$. The value $\varepsilon = 1$ reflects the case known as utilitarian and expresses an indifference towards equality or inequality in the distribution of income. Finally, the maximin case is reflected with values of ε approaching $-\infty$.

The CES utility function such as that used here presents two types of parameters for estimation: Parameter a_i with which each one of the components of the utility function is weighted, and parameter β, which determines the value of the elasticity of substitution, as stated in the following equation:

$$e = \frac{1}{1-\beta}$$

Studies performed in other countries show a wide range of elasticity of substitution values from 0.04 to 2.14. In a recent paper, M. Tuomala (1984) has emphasized the importance of determining the actual value of the elasticity of substitution as accurately as possible, in view of its considerable influence on the value obtained for the utility function. In fact, the difference between the results found by Mirrlees, on the one hand, and Stern, on the other hand, is due to the fact that the former uses a linear logarithmic utility function which assumes an unitary elasticity of substitution value between 0.4 and 0.5. These values, in turn, express values for β between 1.5 and 1. Ahijado (1983) uses a value for elasticity of substitution of about 0.977 for the Spanish case.

On the other hand, as base-point values for the utility function a_i parameters, we will use those estimated by Wales and Woodland[10] for the United States when only income consumed and leisure are involved in the utility function. In such cases, the values $a_1 = 0.6475$ and $a_2 = 0.3525$ adequately reflect the rating that any individual gives to income and leisure, respectively. If we were to introduce public goods as a utility function argument, we would have no information concerning the value of the a_i parameters. However, we will compute the optimal tax rates with public goods using values for a_1, a_2 and a_3 that, by way of sole condition, meet the criteria that income is valued twice as highly as leisure. This valuation, estimated by Wales and Woodland, seems to be fairly realistic. Moreh (1983) describes a method for calculating values for a_1, a_2 and a_3; however, as this is a simulation method that does not use real parameters, it appears to be more appropriate not to use it here.

Finally, we will use a range of values for g (share of expenditure in public goods in the national income) varying from 0.10 to 0.30, which contains the value of g for most industrialized countries.

The procedure devised to calculate the tax rates that maximize the social welfare function differ from the one used by Stern or Tuomala in that we use a sub-optimization process within the problem of general optimization. In fact, given base line values for the parameters a_1, a_2, a_3, g, ε and β, a base line value for t and y is chosen arbitrarily. The value for t is chosen within the interval (0.01 and 0.99). The base line values for t and y0 are substituted into equation (9) to find the expression for the supply of labor that expresses the optimal behavior of the ith

[9] See Stern (1977).

[10] See Wales and Woodland (1979).

individual. After finding the equation for l_i, the value of R is calculated. When this value for R is substituted into equation (11), it is possible to find a new value for y0. If this value for y0 coincides with the base line value, the process proceeds as described later. If the new value for y0 does not coincide with the base line value, the former replaces the latter in the equation for l_i and the process is restarted. This step of the process can be considered as an optimization of the value for y0, thanks to the programming routine used known as Zeroin. Wilkinson[11] and Forsythe et al.[12] give a complete description of the routine. It is an iterative algorithm whose purpose is to seek solutions (zeros) to the equations using the information originally given.

Upon finding a value for y_0 that is compatible with the base of point value for t, we then obtain the equation for y_{di} by substituting y_0 in equation (7). After finding y_{di}, the equation for U_i can be expressed with respect to w_i, substituting the expression for U_i in (10) and integrating it with respect to w_i to obtain the value of the social welfare function. In order to ensure the maximum value for W, we have used the FMIN[13] subroutine. This subroutine can be used to calculate the minimum of an F function or the maximum of a –F function and includes the parabolic interpolation method. The subroutine requires that the values of t be found according to the value obtained by the social welfare function for the immediately preceding value of t. When the social welfare function peaks, the corresponding of value of t is optimal.

If equation (13) above (related to the economic stabilization goal) is introduced as a constraint to optimize the behavior of the public sector, our program would consist of calculating values of t that maximize the social welfare function and that, at the same time, together with the parameter y_0, satisfy this constraint.

Before concluding the description for the computation procedure, we will just point out that the w_i, l_i and U_i functions have been integrated following the sub-routine known as Quancs[14] – an acronym based on the initials of "quadrature", "adaptative", "Newton-Cotes' 8-panel" – based on Newton Cotes' integration method.

4. Numerical Results

This section contains the results obtained using the computation method described in the previous section[15]. We have performed a basic calculation (see Table 1 for results) which will be used as a reference for discussing the remaining results. This basic calculation uses the following hypotheses: 1) goals related with the stabilization policy are not taken into account; 2) the supply of public goods does not affect individual utility and consequently $a_3 = 0$; 3) the ability variable density function parameters considered are those obtained by Lydall, that is, $\mu = -1$ and $\angle = 0.39$; 4) our reference value will be the elasticity of substitution e = 0.4, which will give a reference value for β of –1.5; 5) the reference value for the elasticity of the

[11] See Wilkinson (1967).

[12] See Forsythe (1977).

[13] A more complete description of this sub-routine may be found in Forsythe, G.E. et al. (1977), pp. 179-187.

[14] See Forsythe et al. (1977), pp. 97-103.

[15] The calculations have been made with a Vax computer, model 780, belonging to the Universitat Politècnica de Catalunya.

marginal utility of the income is $\varepsilon = -1$; and 6) the parameters a_1 and a_2[16] are 0.6475 and 0.3525, respectively.

Table 1 provides the optimal taxation results for the hypotheses that have just been described. The first conclusion obtained is that the marginal rates increase is in line with the growth in the level of tax collection required by the public sector. In any case, observe that the maximum value that g can have is 0.3 which, although large, is not a disproportionate value. At higher values for g, it is likely that the marginal tax rates will tend to decrease.

If we let g have a value of 0.12, we can analyze the effects of a variation of the elasticity of the marginal utility of income on the tax rates. It has been verified that, for an acceptable value of the elasticity of substitution such as $e = 0.4$ (or $\beta = -1.5$), the marginal tax rate rises to over 60% when ε tends to minus infinite. Remember that this is a maximin case. Tax rates fall as the preference for strict equality in income level falls. For a value of $\varepsilon = 1$ (utilitarian case) the marginal rate is 27.35%. It is also interesting to verify that the guaranteed income, y_0, increases when the preference for equality increases, and reaches its maximum value when ε tends to minus infinite.

Finally, if we allow β – which determines the elasticity of substitution between income and leisure – to vary freely, the intuitive result is obtained that tax rates increase in line with increases in this parameter or, what amounts to the same thing, as the elasticity of substitution decreases. Also, as the elasticity of substitution decreases, the fixed sum component of the tax, y_0, increases.

It is possible to compare these basic results with those obtained for similar cases in previous computation models. Mirrlees' standard case assumes $\beta = 0$ (unitary elasticity of substitution), $\beta = 0$ and $g = 0$. Atkinson's typical case assumes that e tends to minus infinite, = 0 and R = 0. Stern's significant case assumes $\beta = -1.4$, $\varepsilon = 1$, $g = 0.10$ and $a_2 = 0.3864$. Finally, Tuomala's significant case assumes that $\beta = 1$, $\varepsilon = 1$ and R = 10%. However, both Mirrlees' and Tuomala's model consider a general income tax, as is our case. Consequently, the significant tax rate in those models will probably be the one that will be borne by the individual whose ability level coincides with the mean distribution of abilities among the population.

Upon comparing the results obtained by these orders in the models with ours, a surprising similarity is observed. Mirrlees obtains a representative marginal rate of 33%, while in our model the marginal rate is 30%. For the maximin case, Atkinson obtains a marginal rate of 60% while our model obtains a marginal rate of 61.73%. For the above stated conditions, Stern derives a tax rate of 23.3% while we have obtained a rate of 25.3% for similar conditions. Finally, Tuomala obtains for the maximin case values approaching 60% which coincide, as we have seen before, with our results. The model presented in this paper has sufficient explanatory capacity to include as special cases the results obtained in previous models.

The remaining Tables –2 to 7– show the results obtained by our model when modifications are introduced in some hypotheses or, as in the case in Tables 6 and 7, when we use tax to achieve the stabilization goals.

[16] The values of Y_0 and R, are multiplied by 1000. Their absolute values are not of interest to us as they are measured in units of effectiveness.

The results of Tables 2 and 3 express the optimal values of the marginal tax rate when parameters a_1, a_2 and a_3 of the individual utility function are modified in order to enable the introduction of public goods. The weights used have been chosen on the condition that the income weight is double that of leisure and that the weight allocated to public goods is the least of the three; of course, it is also necessary that the sum of the three be equal to one.[17]

Table 2 shows the results found for $a_1 = 0.53$, $a_2 = 0.27$ and $a_3 = 0.20$. The main difference with respect to the results of Table 1 is that the former, with occasional exceptions, are noticeably lower than the latter. Apart from the logical analytic explanation given by the structure of our model, it is not easy to find an intuitive reason for this result. Thus, Moreh obtains an opposite conclusion: the introduction of public goods increases the marginal tax rates in relation to the previous situation. However, he compares situations which are not comparable (one is derived from his model and the other is derived from Stern's model and, although both models have common features, they do not lead to the same results). Furthermore, Moreh compares both models and the only difference he sees in them is that his model includes the cost of supplying public goods as a part of the revenue constraint on the public sector and Stern does not (however, this point is not exactly true), but he forgets that public goods are introduced, in his model, as an argument in the utility function while Stern does not consider this possibility. In short, to be able to assess it, we would need to calculate, for the same model, marginal rates before and after introducing public goods and then look for differences. The main result we have found is that marginal rates are lower when public goods are introduced into the utility function.

In order to observe the sensitivity of the results obtained – when public goods are taken into account – to the parameters a_1, a_2 and a_3, we have recalculated the optimal values when $a_1 = 0.60$, $a_2 = 0.30$ and $a_3 = 0.10$. In this case, it is observed that the tax rates are slightly higher than those found in Table 2, although they are still below those that appear in Table 1. It can thus be stated that, in general, the lower the weight allocated to public goods in the utility function, the higher the marginal tax rates will be.

Tables 4 and 5 show the values found for a particular simulation: we have introduced a standard deviation of the distribution of abilities higher than the initial deviation (1.00 versus 0.39), which means that this variable is more widely spread and, therefore, the inequality in the ability to generate income between individuals is greater. As expected, the marginal rates are much higher than when $\angle = 0.39$, which is a logical consequence of the tax's redistributional emphasis.

In particular, in Table 4 we have calculated utility functions with weighting values equal to those of Table 1, i.e., $a_1 = 0.6475$, $a_2 = 0.3525$ and $a_3 = 0$. In Table 5, the calculations have been performed taking into account the effects of public goods on the individual utility, using the parameters of Table 2, i.e., $a_1 = 0.53$, $a_2 = 0.27$ and $a_3 = 0.20$. This case confirms the trend we have observed on the results of Tables 2 and 3, that is, when public goods are taken into account in the ability function, the marginal rates generally fall with respect to the prior situation when they were not taken into account.

[17] In order to simplify the computation method, we have assumed in Table 3 that $\delta P / \delta l_i = 0$. What it means is that labor supply has no effects on public goods supply, in the short term.

We move now into Tables 6 and 7. The results obtained reflect the introduction of a major additional factor: the public sector constraint. If we analyze Tables 6 and 7 with more detail, and compare them with Tables 1 and 2, respectively, that is to say, compare the results with the stabilization constraint with those obtained without it, we can observe that the range of variations for each one of the six simulations computed has narrowed. The only exception is the first computation showed in Table 1 (Epsilon = –1, Beta = –1.5), which, in comparison with Table 6, has roughly the same range of variation. For instance, the second computation in Table 1 (g = 0,12 and β = –1.5) yields the interval (0.6173, 0.1000) for t, while under the same hypothesis, in Table 6 we have obtained (0.5773, 0.2500). The same trace can be observed with the rest of the simulations.

So, we can assert that when the government designs the optimal income tax with some kind of stabilization goal, the consequent constraint has the effect of reducing the highest marginal tax rate and increasing the lowest, narrowing the variation range of the optimal tax rates. This result is probably consistent with the idea of giving the tax system as much flexibility as possible, in the classical meaning of flexibility as a property of a good tax. Anyway, this is the main conclusion of this paper, and it is worth incorporating into optimal tax literature.

5. Final Notes

The computational model presented in this section has obvious limitations. However, as we have seen, it enables comparisons to be made between the optimal marginal rates when a stabilization constraint is included and when it is not included, between the optimal rates when public goods are included in the utility function and when they are not included, and finally, between the optimal tax rate values when discretional variations are introduced in all the model parameters.

In view of these results, is it possible to suggest concrete reforms for any tax system? No doubt a number could be formulated such as the need to reduce the number of income levels taxed at different marginal rates. However, one must bear firmly in mind the purpose of this kind of literature. The aim is not to provide a definite numerical answer to the question, "how progressive should the income tax be? ... the purpose is rather to explore the implications of different beliefs about how the world works or about how governments should behave" (Atkinson and Stliglitz, 1980, pp. 422-423).

The main result presented is that related to the values of marginal tax rates when we introduce the stabilization constraint. In this case, we have shown that the variation range of the optimal tax rates has narrowed for almost all cases. This is a new result in the optimal taxation theory.

Table 1

BASE-POINT VALUES

Epsilon=-1; Beta=-1.5

Mu=-1; Sigma=0.39; a1=0.6475; a2=0.3525; a3=0

variable:

g	t	Y0	R	W
0.1	0.2138	0.0797	0.2541	7.0748
0.15	0.2490	0.0767	0.2565	7.0411
0.17	0.2661	0.0750	0.2578	7.0234
0.20	0.2841	0.0736	0.2590	7.0052
0.22	0.3024	0.0722	0.2603	6.9864
0.25	0.3189	0.0703	0.2617	6.9669
0.28	0.3362	0.0687	0.2631	6.9467
0.30	0.3545	0.0673	0.2644	6.9258

BASE-POINT VALUES

g=0.12; Beta=-1.5

Mu=-1; Sigma=0.39; a1=0.6475; a2=0.3525; a3=0

variable:

Epsilon	t	Y0	R	W
-oo	0.6173	0.1217	0.2353	-235.2779
-2	0.4434	0.1078	0.2432	-8.0746
-1.5	0.4233	0.1038	0.2452	-5.3295
-1	0.3988	0.0986	0.2474	-3.9753
-0.5	0.3679	0.0919	0.2500	-3.9749
-0.01	0.3265	0.0826	0.2532	-1.3759
0.01	0.3238	0.0820	0.2534	98.6431
1	0.2535	0.0456	0.2631	0.2600
2	0.1000	0.0000	0.2716	0.0361

BASE-POINT VALUES

g=0.12; Epsilon=-1

Mu=-1; Sigma=0.39; a1=0.6475; a2=0.3525; a3=0

variable:

Beta	t	Y0	R	W
-2.25	0.5689	0.1076	0.2632	7.0729
-2	0.4929	0.1029	0.2618	7.0729
-1.75	0.4765	0.0980	0.2603	7.0731
-1.50	0.4572	0.0924	0.2586	7.0734
-1.25	0.4370	0.0864	0.2565	7.0739
-1	0.4138	0.0797	0.2541	7.0748
-0.75	0.3315	0.0731	0.2509	7.0761
-0.50	0.2541	0.0652	0.2469	7.0783
-0.25	0.1961	0.0569	0.2412	7.0819
-0.1	0.1881	0.0516	0.2366	7.0852

Table 2

BASE-POINT VALUES

Epsilon=-1; Beta=-1.5

Mu=-1; Sigma=0.39; a1=0.53; a2=0.27; a3=0.20

variable:

g	t	Y0	R	W
0.1	0.0900	0.0000	0.2715	6.2801
0.15	0.1000	0.0000	0.2751	6.4808
0.17	0.1100	0.0000	0.2751	6.5509
0.20	0.1350	0.0000	0.2751	6.6087
0.22	0.1870	0.0000	0.2751	6.6571
0.25	0.2040	0.0151	0.2798	6.6516
0.28	0.2485	0.0206	0.2800	6.6746
0.30	0.1000	0.0000	0.2751	6.7654

BASE-POINT VALUES

g=0.12; Beta=-1.5

Mu=-1; Sigma=0.39; a1=0.53; a2=0.27; a3=0.20

variable:

Epsilon	t	Y0	R	W
-oo	0.4403	0.0110	0.2733	-29305.91
-2	0.3300	0.0000	0.2751	-56.7425
-1.5	0.2500	0.0000	0.2751	-23.1076
-1	0.2000	0.0000	0.2751	-10.6091
-0.5	0.1700	0.0000	0.2751	-6.5078
-0.01	0.1422	0.0000	0.2751	-2.3858
0.01	0.1312	0.0000	0.2751	97.6698
1	0.1176	0.0000	0.2751	0.0949
2	0.1000	0.0000	0.2751	0.0045

BASE-POINT VALUES

g=0.12; Epsilon=-1

Mu=-1; Sigma=0.39; a1=0.6475; a2=0.3525; a3=0

variable:

Beta	t	Y0	R	W
-2.25	0.4973	0.0000	0.2787	5.8980
-2	0.4580	0.0000	0.2782	5.9512
-1.75	0.4391	0.0000	0.2777	6.0149
-1.50	0.4000	0.0000	0.2770	6.0906
-1.25	0.3200	0.0000	0.2762	6.1793
-1	0.2842	0.0000	0.2751	6.2801
-0.75	0.2333	0.0000	0.2637	6.3891
-0.50	0.1631	0.0000	0.2717	6.4997
-0.25	0.1484	0.0001	0.2689	6.6039
-0.1	0.1189	0.0050	0.2648	6.6610

Table 3

BASE-POINT VALUES

Epsilon=-1; Beta=-1.5

Mu=-1; Sigma=0.39; a1=0.60; a2=0.30; a3=0.1

variable:

g	t	Y0	B	W
0.1	0.1004	0.0001	0.2752	6.5638
0.15	0.2299	0.0219	0.2738	6.6939
0.17	0.2547	0.0299	0.2733	6.7314
0.20	0.2811	0.0358	0.2732	6.7577
0.22	0.2918	0.0401	0.2734	6.7756
0.25	0.3197	0.0437	0.2736	6.7870
0.28	0.3329	0.0460	0.2741	6.7932
0.30	0.1000	0.0000	0.2752	6.8952

BASE-POINT VALUES

g=0.12; Beta=-1.5

Mu=-1; Sigma=0.39; a1=0.60; a2=0.30; a3=0.1

variable:

Epsilon	t	Y0	B	W
-oo	0.4418	0.0635	0.2628	-4990.53
-2	0.3324	0.0356	0.2690	-27.6863
-1.5	0.2048	0.0283	0.2704	-13.4576
-1	0.1741	0.0201	0.2719	-7.3842
-0.5	0.1363	0.0099	0.2736	-5.4230
-0.01	0.1201	0.0000	0.2752	-112.010
0.01	0.1125	0.0007	0.2751	98.0295
1	0.1030	0.0000	0.2752	0.1379
2	0.1010	0.0000	0.2752	0.0096

BASE-POINT VALUES

g=0.12; Epsilon=-1

Mu=-1; Sigma=0.39; a1=0.60; a2=0.30; a3=0.1

variable:

Beta	t	Y0	B	W
-2.25	0.5000	0.0000	0.2788	6.1066
-2	0.4980	0.0038	0.2783	6.1796
-1.75	0.4870	0.0027	0.2777	6.2636
-1.50	0.4000	0.0023	0.2771	6.3579
-1.25	0.3370	0.0002	0.2762	6.4598
-1	0.3024	0.0011	0.2752	6.5638
-0.75	0.2422	0.0114	0.2716	6.6628
-0.50	0.1880	0.0234	0.2662	6.7508
-0.25	0.1669	0.0303	0.2594	6.8245
-0.1	0.2246	0.0317	0.2546	6.8619

Table 4

BASE-POINT VALUES

Epsilon=-1; Beta=-1.5

Mu=-1; Sigma=1; a1=0.6475; a2=0.3525; a3=0

variable:

____g____	____t____	____YO___	____B____	____W____
0.1	0.2886	0.1782	0.3027	7.2435
0.15	0.3071	0.1709	0.3069	7.2117
0.17	0.3163	0.1673	0.3090	7.1950
0.20	0.3258	0.1636	0.3112	7.1777
0.22	0.3356	0.1600	0.3133	7.1599
0.25	0.3543	0.1561	0.3158	7.1414
0.28	0.3839	0.1523	0.3181	7.1223
0.30	0.4634	0.1485	0.3205	7.1024

BASE-POINT VALUES

g=0.12; Beta=-1.5

Mu=-1; Sigma=1 a1=0.6475; a2=0.3525; a3=0

variable:

Epsilon	____t____	____YO___	____B____	____W____
-00	0.6776	0.1920	0.2833	-91.389
-2	0.6491	0.1885	0.2904	-5.5313
-1.5	0.6402	0.1872	0.2924	-4.0085
-1	0.6293	0.1855	0.2948	-3.2839
-0.5	0.5151	0.1831	0.2977	-3.6094
-0.01	0.4972	0.1799	0.3012	-101.17
0.01	0.4937	0.1792	0.3018	98.836
1	0.3092	0.1605	0.3151	0.3197
2	0.2100	0.0000	0.3538	0.0638

BASE-POINT VALUES

g=0.12; Epsilon=-1

Mu=-1; Sigma=1; a1=0.6475; a2=0.3525; a3=0

variable:

__Beta___	____t____	____YO___	____B____	____W____
-2.25	0.5838	0.2153	0.3149	7.2454
-2	0.4693	0.2092	0.3126	7.2440
-1.75	0.4529	0.2026	0.3103	7.2429
-1.50	0.4337	0.1952	0.3079	7.2422
-1.25	0.4127	0.1871	0.3054	7.2423
-1	0.3886	0.1782	0.3027	7.2435
-0.75	0.3620	0.1685	0.2998	7.2466
-0.50	0.3312	0.1575	0.2966	7.2530
-0.25	0.2976	0.1456	0.2925	7.2651
-0.1	0.2555	0.1376	0.2895	7.2773

Table 5

BASE-POINT VALUES

Epsilon=-1; Beta=-1.5

Mu=-1; Sigma=1; a1=0.53; a2=0.27; a3=0.20

variable: g	t	YO	R	W
0.1	0.1429	0.1162	0.3391	6.4247
0.15	0.2358	0.1301	0.3373	6.6212
0.17	0.2716	0.1336	0.3370	6.6870
0.20	0.2917	0.1354	0.3372	6.7389
0.22	0.3293	0.1365	0.3375	6.7801
0.25	0.3524	0.1361	0.3384	6.8128
0.28	0.3735	0.1353	0.3395	6.8385
0.30	0.3943	0.1342	0.3405	6.8583

BASE-POINT VALUES

g=0.12; Beta=-1.5

Mu=-1; Sigma=1; a1=0.53; a2=0.27; a3=0.20

variable: Epsilon	t	YO	R	W
-oo	0.6310	0.1425	0.3307	-11379.94
-2	0.5908	0.1308	0.3348	-39.0096
-1.5	0.5822	0.1282	0.3356	-17.4406
-1	0.5119	0.1251	0.3365	-8.7910
-0.5	0.4810	0.1218	0.3375	-5.9221
-0.01	0.4450	0.1169	0.3389	-102.192
0.01	0.4434	0.1164	0.3390	9.7885
1	0.3153	0.1076	0.3413	0.1151
2	0.2686	0.0926	0.3448	0.0067

BASE-POINT VALUES

g=0.12; Epsilon=-1

Mu=-1; Sigma=1; a1=0.53; a2=0.27; a3=0.20

variable: Beta	t	YO	R	W
-2.25	0.5799	0.0948	0.3387	6.0326
-2	0.5436	0.0962	0.3393	6.0876
-1.75	0.4950	0.1002	0.3398	6.1529
-1.50	0.4090	0.1050	0.3400	6.2303
-1.25	0.3470	0.1111	0.3397	6.3210
-1	0.3229	0.1162	0.3391	6.4247
-0.75	0.2898	0.1214	0.3375	6.5388
-0.50	0.2720	0.1246	0.3350	6.6577
-0.25	0.2366	0.1249	0.3317	6.7749
-0.1	0.2057	0.1236	0.3290	6.8430

Table 6

INCLUSION OF STABILIZATION CONSTRAINT

BASE-POINT VALUES

Epsilon=-1; Beta=-1.5

Mu=-1; Sigma=0.39; a1=0.6475; a2=0.3525; a3=0

variable: g	t	YO	R	W
0.1	0.2141	0.0798	0.2541	7.0748
0.15	0.2298	0.0769	0.2565	7.0411
0.17	0.2475	0.0754	0.2577	7.0234
0.20	0.2643	0.0736	0.2590	7.0052
0.22	0.2702	0.0716	0.2604	6.9864
0.25	0.2987	0.0703	0.2617	6.9669
0.28	0.3364	0.0687	0.2631	6.9467
0.30	0.3533	0.0670	0.2645	6.9258

BASE-POINT VALUES

g=0.12; Beta=-1.5

Mu=-1; Sigma=0.39; a1=0.6475; a2=0.3525; a3=0

variable: Epsilon	t	YO	R	W
-00	0.5773	0.1217	0.2353	-235.28
-2	0.4435	0.1079	0.2432	-8.0746
-1.5	0.4031	0.1037	0.2452	-5.3295
-1	0.3889	0.0986	0.2473	-3.9753
-0.5	0.3579	0.0919	0.2500	-3.9749
-0.01	0.3170	0.0828	0.2531	-1.0137
0.01	0.2783	0.0831	0.2530	9.8643
1	0.2601	0.0523	0.2615	0.2600
2	0.2500	0.0523	0.2615	0.0354

BASE-POINT VALUES

g=0.12; Epsilon=-1

Mu=-1; Sigma=0.39; a1=0.6475; a2=0.3525; a3=0

variable: Beta	t	YO	R	W
-2.25	0.5084	0.1075	0.2632	7.0729
-2	0.4917	0.1026	0.2619	7.0729
-1.75	0.4752	0.0977	0.2604	7.0731
-1.50	0.4569	0.0923	0.2586	7.0734
-1.25	0.4368	0.0864	0.2565	7.0739
-1	0.4141	0.0798	0.2541	7.0748
-0.75	0.3908	0.0729	0.2509	7.0761
-0.50	0.3648	0.0653	0.2468	7.0783
-0.25	0.3357	0.0568	0.2413	7.0819
-0.1	0.3170	0.0513	0.2367	7.0852

Table 7

INCLUSION OF STABILIZATION CONSTRAINT

BASE-POINT VALUES

Epsilon=-1; Beta=-1.5

Mu=-1; Sigma=0.39; a1=0.53; a2=0.27; a3=0.20

variable:

g	t	Y0	B	W
0.1	0.1940	0.0530	0.2652	6.2669
0.15	0.2018	0.0405	0.2700	6.4658
0.17	0.2134	0.0340	0.2724	6.5313
0.20	0.2326	0.0275	0.2749	6.5821
0.22	0.2537	0.0208	0.2775	6.6214
0.25	0.2699	0.0150	0.2798	6.6516
0.28	0.2783	0.0213	0.2799	6.6746
0.30	0.2923	0.0258	0.2802	6.6918

BASE-POINT VALUES

g=0.12; Beta=-1.5

Mu=-1; Sigma=0.39; a1=0.53; a2=0.27; a3=0.20

variable:

Epsilon	t	Y0	B	W
-oo	0.5013	0.0530	0.2652	-30318.17
-2	0.4610	0.0530	0.2652	-58.2869
-1.5	0.4099	0.0530	0.2652	-23.6136
-1	0.3813	0.0530	0.2652	-10.7741
-0.5	0.3743	0.0530	0.2652	-6.5613
-0.01	0.3413	0.0530	0.2652	-2.4035
0.01	0.3096	0.0530	0.2652	-9.7652
1	0.2914	0.0530	0.2652	0.0932
2	0.2413	0.0530	0.2652	0.0043

BASE-POINT VALUES

g=0.12; Epsilon=-1

Mu=-1; Sigma=0.39; a1=0.53; a2=0.27; a3=0.20

variable:

Beta	t	Y0	B	W
-2.25	0.4000	0.0547	0.2734	5.8860
-2	0.3843	0.0544	0.2723	5.9385
-1.75	0.3632	0.0542	0.2711	6.0015
-1.50	0.3148	0.0539	0.2695	6.0768
-1.25	0.3000	0.0535	0.2676	6.1655
-1	0.2899	0.0530	0.2652	6.2669
-0.75	0.2708	0.0524	0.2620	6.3772
-0.50	0.2477	0.0515	0.2576	6.4896
-0.25	0.2286	0.0502	0.2513	6.5959
-0.1	0.2135	0.0492	0.2460	6.6541

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